Advanced Data Structures

Intro
Union-Find

Today

- Overview of topic
- Syllabus :
- First data structure

Overview: Why?

Data structures are useful!

Often just use existing ones - but understanding trade-offs is key.

I'm assuming you've had an (intro) data structures couse, as well as an algorithms course.

Reason: Beyond those "simpler" intro ones, things get tricky.

I want to emphasize:

- simple + elegant
- powerful
- useful

Next: Syllabus!

(Boring but necessary)

First data structure: Union-Find

(Have any of you already seen it?)

Goal: Keep track of a set of objects that is divided into some # of disjoint subsets, where subsets may be merged. Want to (quickly) answer queries about 2 objects being in same subset (or partition).

Why?

- Introduced in '61 by Arden, Galler & Graham, to track variables & testing equivalence. (Needed in Fortran.)

- Later: Minimum spanning trees - grow disjoint forest, until all in one tree

Formally: 3 operations:

makeSet(x): take an item & create a one element set for it

find(x): return "canonical" element of set containing x

union(x, y): Assuming that $x \neq y$, form a new set that is the union of the 2 sets holding x & y, destroying the 2 old sets. (Also selects & returns a canonical element for new set)

How to implement?

- certainly use existing DS.

Table:

Make an array/table with an entry for each element, & label with subset id.

Ex: makeset(x)
makeset(y)
makeset(z)
union(x, z)
makeset(a)
makeset(b)
union(a, x)
union(b, y)
makeset(c)

Table:

x	1
y	2
z	~~3~~ 1
a	~~3~~ 1
b	~~4~~ 2
c	3

Runtime?

makeset : $O(1)$

find : $O(1)$

union : $O(n)$

So tradeoff w/ this approach:

Bad if many unions.

Better: Use trees!

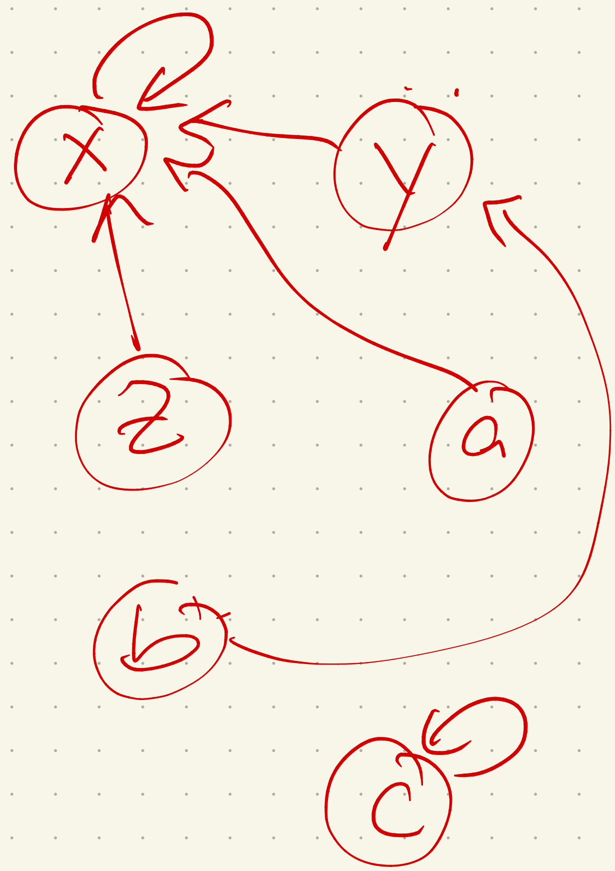
(Galler + Fisher, 1964)

Each set will be a rooted tree, where elements are in the tree & the root is the canonical element.

So each element has a pointer to its parent (& root points to itself)

Ex:

makeset(x) ✓
makeset(y) ✓
makeset(z) ✓
union(x,z) ✓
makeset(a) ✓
makeset(b) ✓
union(a, x)
union(b, y)
makeset(c)
union(z, b)



Then: makeset(x):

create a node w/ value x, + points its pointer to itself

find(x):

travel up the the parent pointer of x, until it points to itself

union(x, y):

combine 2 trees into a single tree by making one of the roots a child of the other root