


CS2100

BST - recap
AVL trees



Outline: -reading assignments this week

- HW due at end of week (partner OK, via git)
- No lab this week, normal lecture instead (all week)
- Next week:

- Sub on Wed & Fri.
- Lab (w/ other sub) on Thursday (due on Sunday)
- HW (on paper) will be due of Monday 4/8 (no extensions)
- Review session 4/8, test on Wed. 4/10 (with sub, but I'll write the exam)
- Then, lab with me on 4/11

Last time: BSTs.

- Code posted

- HW: add remove

(submit readme + test file)

Runtime: (worst case)

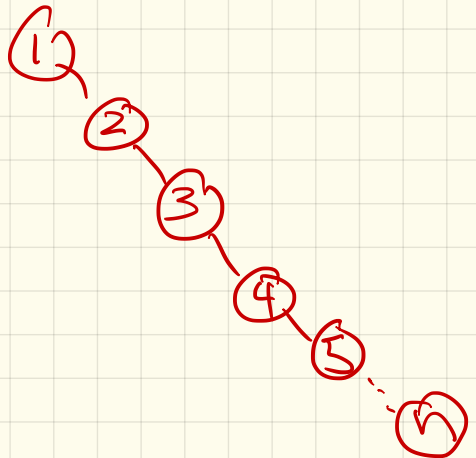
• find

• insert

• remove

$$O(n) = O(h)$$

1, 2, 3, 4, 5, ... n



Balanced BSTs

Many kinds:

- Red-black trees: $\approx 1.39 \lceil \log n \rceil$

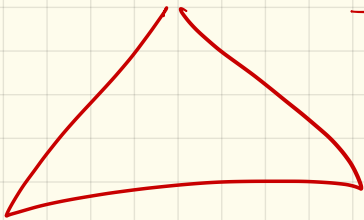
- Splay trees: randomized/expected

- AVL trees $h \leq 2 \lceil \log_2 n \rceil$

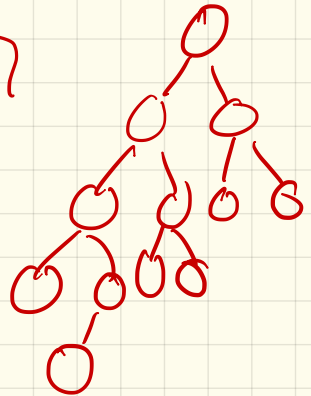
⋮

Goal of them all:

keep the height of
tree $\approx O(\log n)$



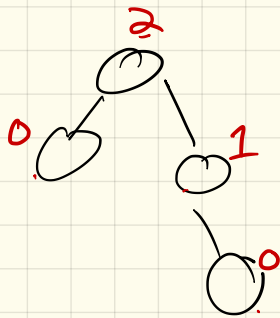
$\lceil \log n \rceil$



AVL trees:

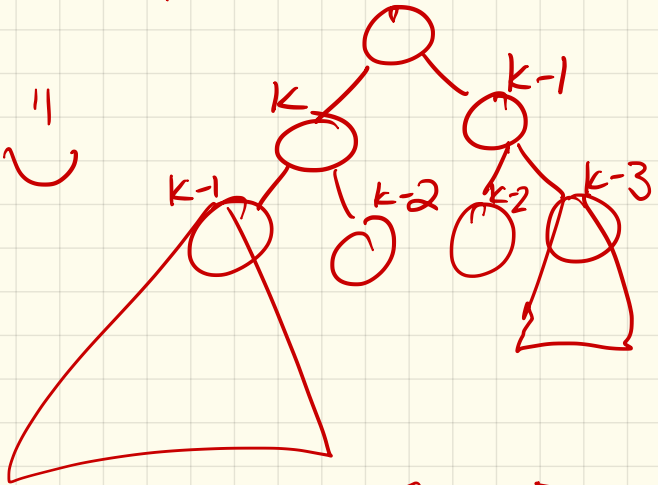
Height balance property:

For every node x in T ,
the heights of x 's
children differ by
at most 1.

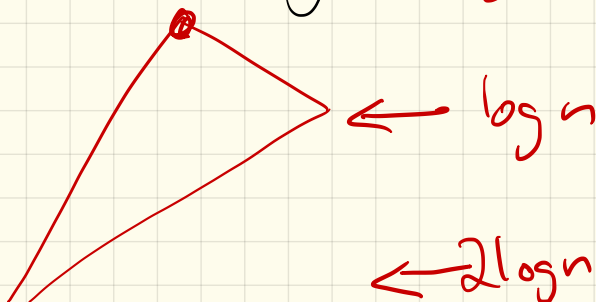


"

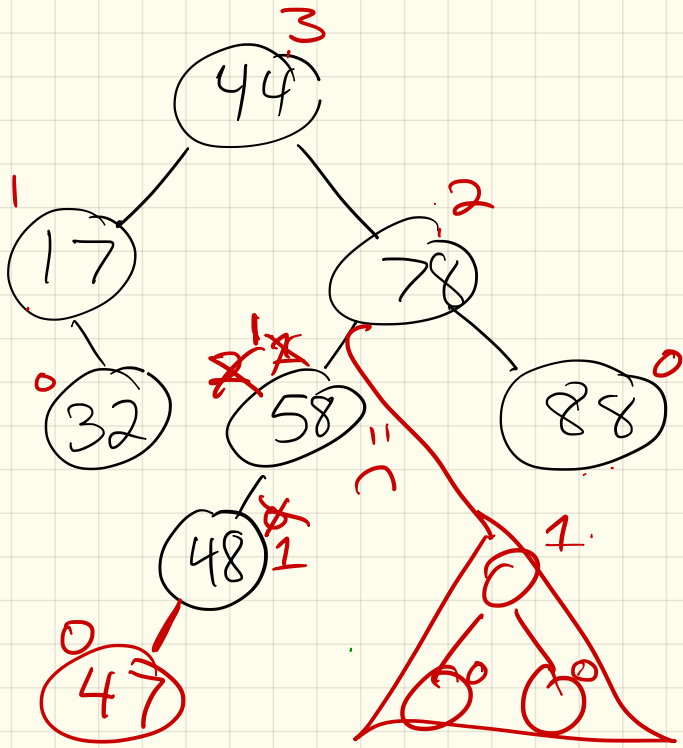
How bad:



$$\Rightarrow \text{max height} \leq 2 \lceil \log_2 n \rceil$$



Ex:



Now: how can we mess this up?

Ex: insert (47).

update heights:

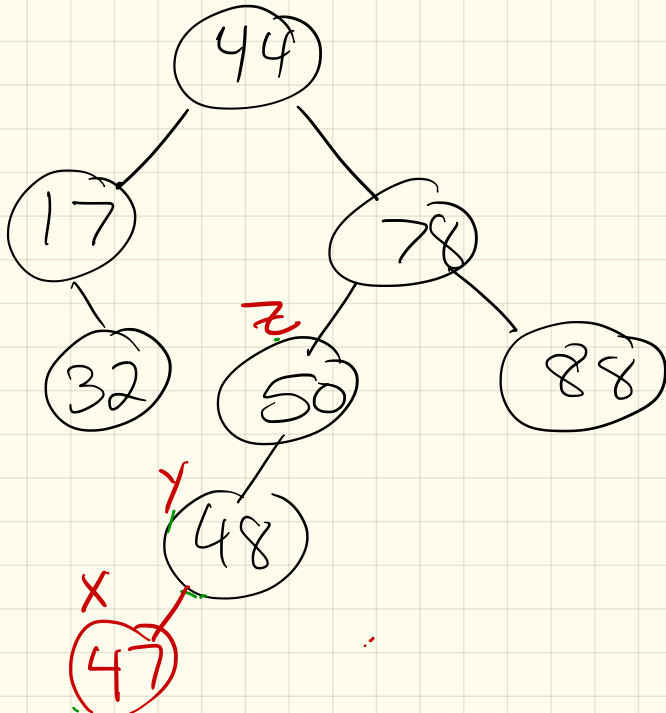
Fix it! How? → rotate (pivot)

Consider the lowest node
which fails the Height-Balance
property.
↳ call this z

Let y be child w/ bigger
height.

Let x be grandchild w/ larger
height.

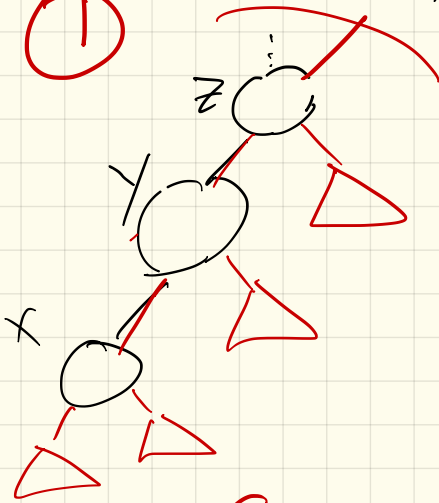
Fix!



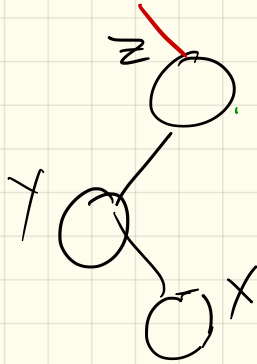
So: algorithm to insert:

- First run BST insert
- Then find lowest unbalanced node z & deeper child/grand child.

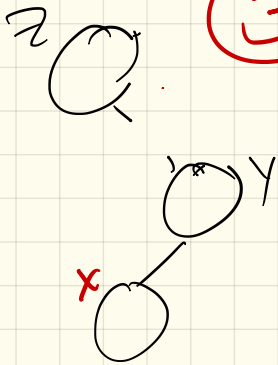
①



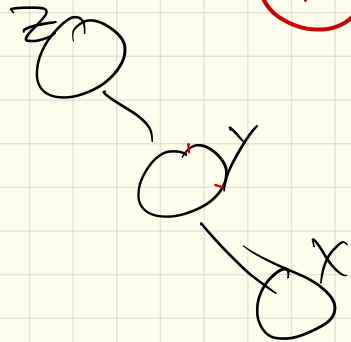
②



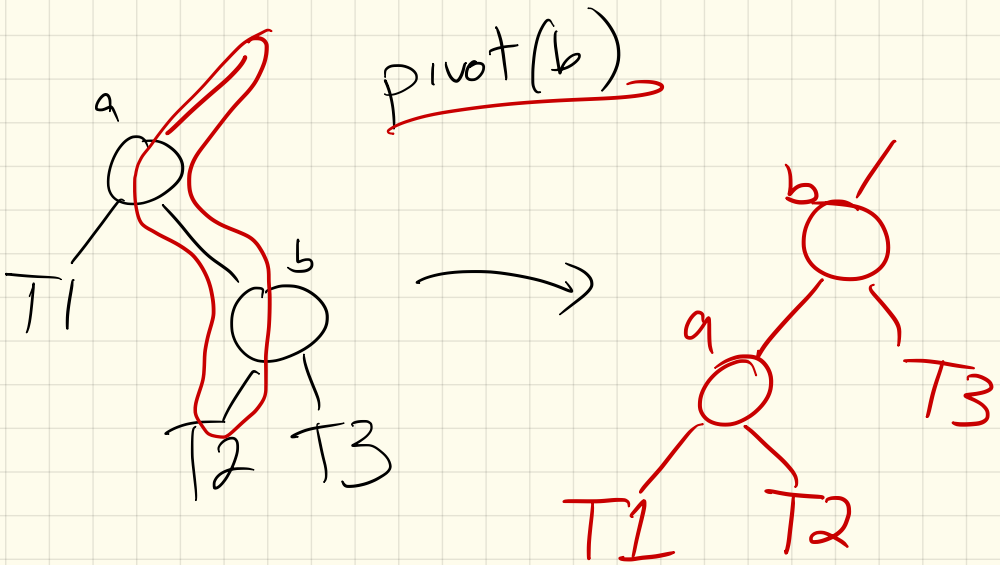
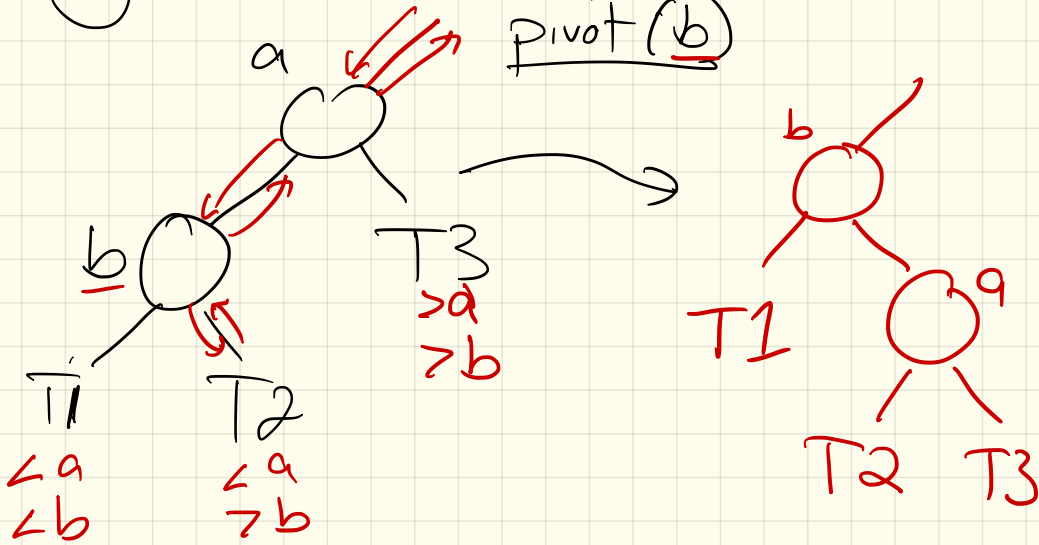
③



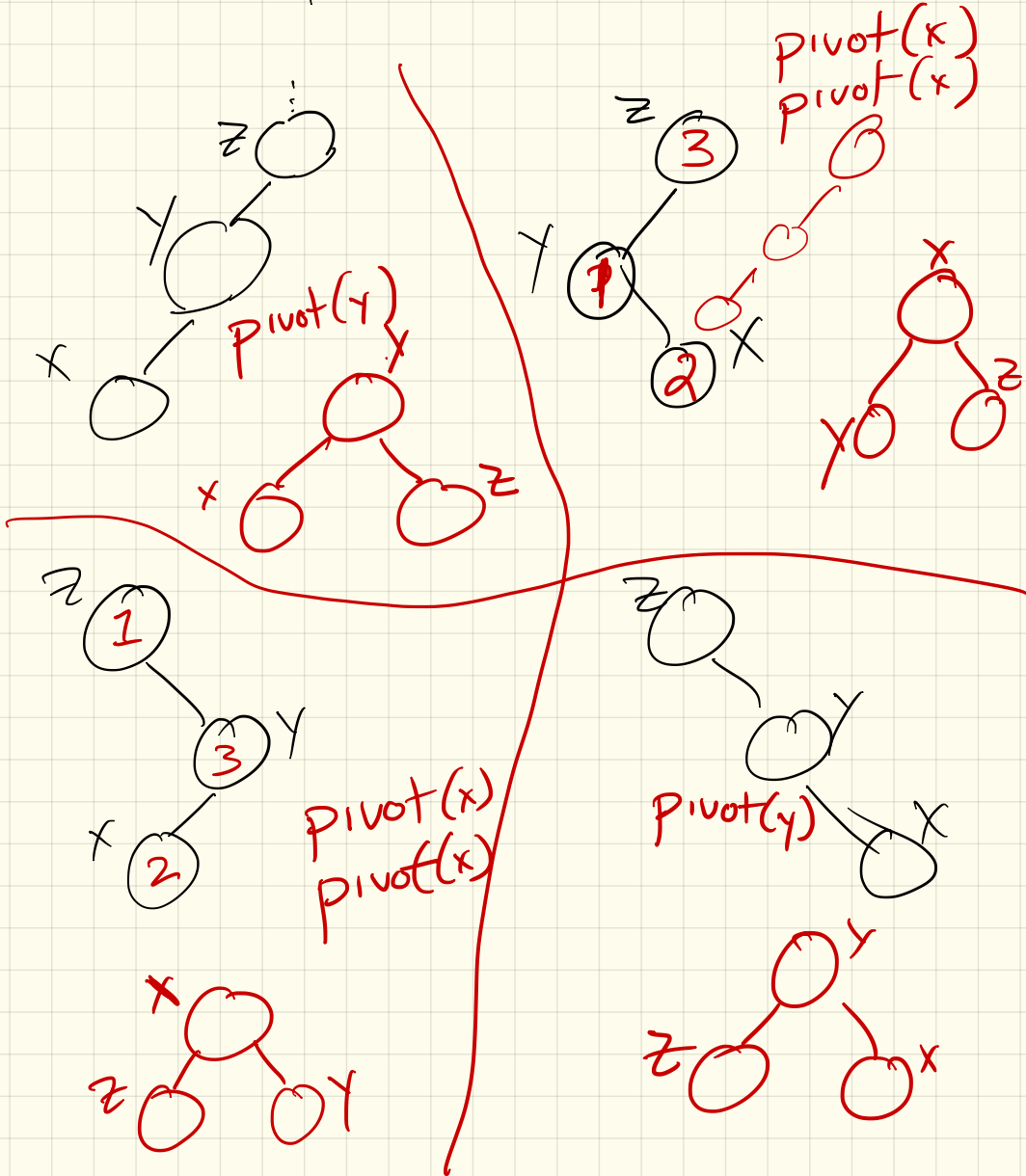
④



Key operation : pivot (in BT)

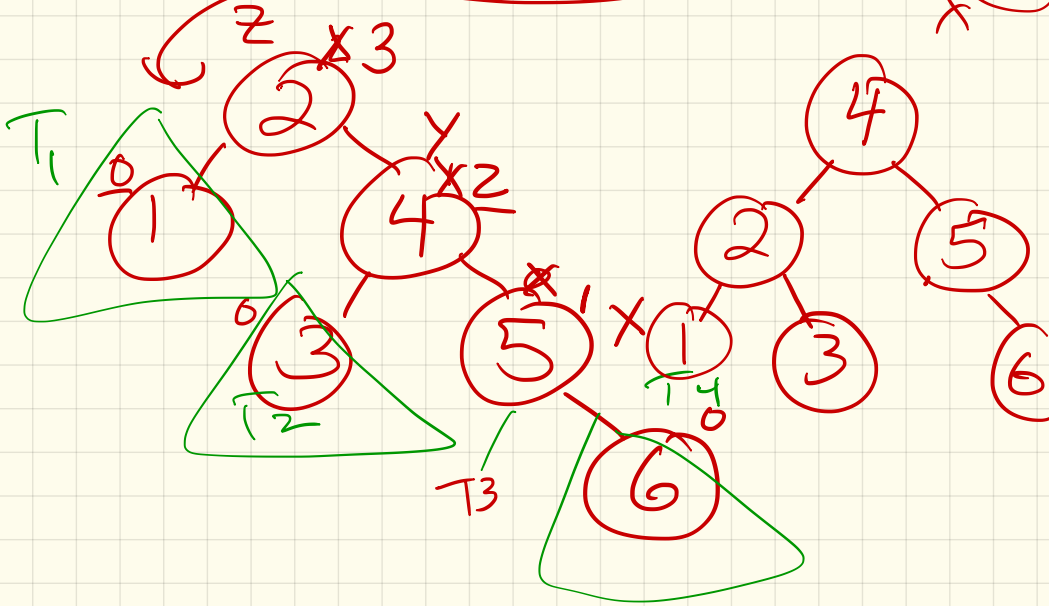
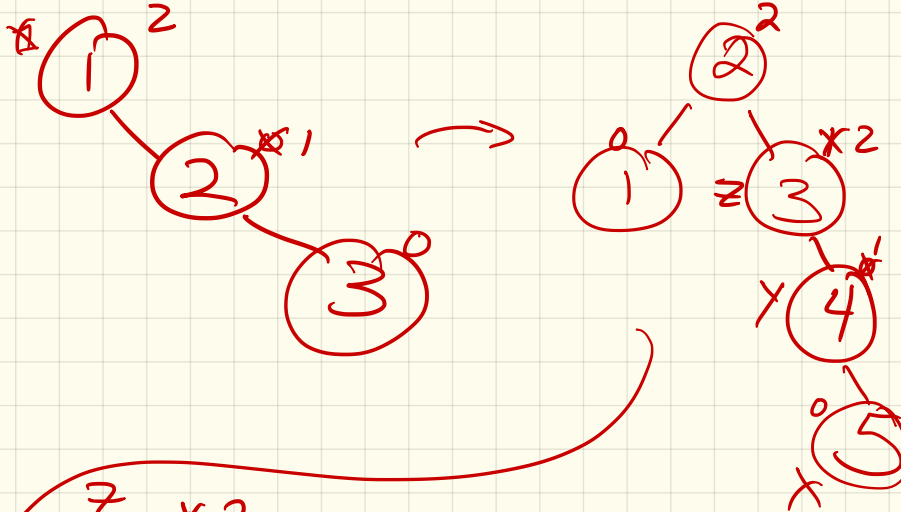


Then: implement w/ pivot!



Bigger example:

Insert: 1, 2, 3, 4, 5, 6, 7



Our code: Need to write:

- height value : in aux
- way to recalculate height
- actual insert & rotates
(pivot)

Implementations:

in BT.h

↳ pivot

In AVLTree.h

↳ insert & height
calculation