CS 314 - Lecture 2

Note Title 1/13/2010

Announcements

- HW0 - due Wed. at <u>start</u> of class!
  (Must be done individually)
- Some helpful links were added to the web page
- Office Hours: Monday 10-11:30
  Thursday 1-2

$F_n = n^{th}$ Fib. #

$F_0 = 0$

$F_1 = 1$

$\forall n > 1, \quad F_n = F_{n-1} + F_{n-2}$

$\lg^{1000} n = (\log_2 n)^{1000}$

$\log_3 n = O(\log_{10} n) \rightarrow O(\log n)$

$\ln n \rightarrow \log_e n$

Last time – big-O

Ex: $n = O(n^2)$

Ex 1: How do 2^n and n^n compare?

$\boxed{2^n = O(n^n)}$: $\lg 2^n$ $\lg n^n$

$\cancel{n} \cdot \lg 2 \leq \cancel{n} \lg n$ for $n \geq 2$

→ $\forall n > 2$, $2 < n$

so $2^x < n^x$

set $N_0 = 3$

and $c = 1$

$\forall n > N_0$, $2^n < 1 \cdot n^n$

$2^n << n^n$

is $n^n << 2^n$? → NO

need $c > 1$ and N_0 s.t. $\forall n > N_0$, $n^n < c \cdot 2^n$

Ex 2: How do $\underbrace{2^{\log_2 n}}_{= n}$ and n^2 compare?

Induction: Recursion's evil twin

(AKA your new best friend)

A method of proving a statement which relies upon smaller values already being shown.

Can think of this as automating a proof:

- Show it holds for base case(s)!
- If holds for values $< n$, then holds for n also.

Ex: $$\underbrace{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}} = O(n^2)$$

Base case: $n=1$ $\qquad \sum_{i=1}^{1} i = 1 \leftarrow$ LHS

Same ✓

RHS $= \frac{1(2)}{2} = 1$

Ind Hyp: $\forall$ values $k<n$, $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

Ind. Step: $$\sum_{i=1}^{n} i = \left(\sum_{i=1}^{n-1} i\right) + n$$

apply IH $$= \frac{(n-1)(n\cancel{-1+1})}{2} + n = \frac{n^2-n}{2} + n$$

$$= \frac{n^2-n+2n}{2} = \frac{n^2+n}{2} = \frac{n(n+1)}{2} \; \blacksquare$$

Ex: $\sum_{j=1}^{n} (2j-1) = n^2$

Base case: $n=1$: $\sum_{j=1}^{1} (2j-1) = 2\cdot 1 - 1 = 1$

$n^2 = 1^2 = 1$ ✓

Ind Hyp: $\forall k < n$, $\sum_{j=1}^{k} (2j-1) = k^2$

Ind Step: $\sum_{j=1}^{n} (2j-1) = \sum_{j=1}^{n-1} (2j-1) + (2n-1)$

apply IH $= (n-1)^2 + (2n-1) = n^2 - 2n + 1 + 2n - 1 = n^2$ ∎

For any induction proof

Need ~~3~~ 4 things:

- know what we are inducting on!
- base case(s)
- Inductive Hypothesis
- Inductive step

Show that $2^n > n^2$ if $n > 4$.

(induction on n)

Base Case: $n = 5$

$2^5 = 32$ ✓

$5^2 = 25$

Ind Hyp: $\forall k < n,\ 2^k > k^2$

→ Ind Step: $2^n = 2^{n-1} \cdot 2$

by IH $> (n-1)^2 \cdot 2 = 2n^2 - 4n + 2$

$= n^2 + (n^2 - 4n + 2)$

$x < y$

then (if $c > 0$)

$cx < cy$

$2^n > n^2$

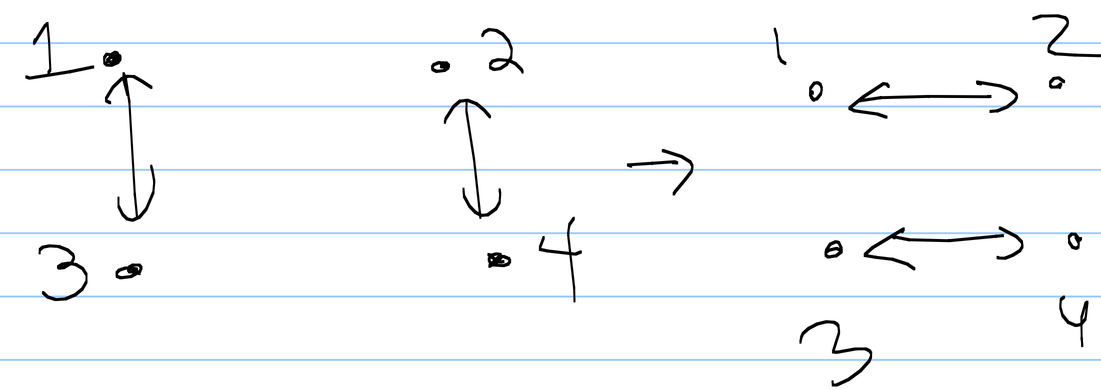
The Gossip Problem

- There are n people, each of which knows a unique secret.
- Every time 2 people call each other, they tell each other all the secrets they currently know.
- How many phone calls are needed before everyone knows all n secrets?

Thm: If $n \geq 4$, then $2n-4$ calls suffice.

Proof by induction: induct on # of people

Base case: $n=4$ Need all 4 people to know all 4 secrets in $2 \cdot 4 - 4 = 4$ calls



<u>pf cont</u>: <u>Ind Hyp</u> For $4 \le k < n$ people, can use $2k - 4$ calls.

<u>Ind step</u>: n people

$n-1$ people $\to$ $2(n-1) - 4$ calls
$= 2n - 6$ calls

Have n call $n-1$.
Apply IH $\to$ $2(n-1) - 4$ calls,
Have $n-1$ call n.

$\blacksquare$

Show that any set of n items has exactly 2^n possible subsets (counting the empty set).

Note about HW:

Some times we need more than 1 base case!

Why?

n

$\hookrightarrow n-3$

$n-5$