CS 314 - Algorithms

Note Title 1/11/2010

Announcements

- Syllabus posted
- HW0 is out - due Wed.

Preregs

- Discrete Math (or equivalent)

- Data Structures

What is an algorithm?

- Problem solving
- A set of steps to solve a problem.
- Transform data into information.
- Finite set of steps . . .

What is a program?

- Implementation of an algorithm.
- May combine algorithms.
- Program expresses algorithm.
- Syntax matters!

What is the difference?

Algorithms are <u>much</u> more general.

Here, we care about algorithms.

3 parts to every question:

1. Algorithm
2. Run time analysis
3. Proof of Correctness

This week: discrete math boot camp!

What did you learn in 135 (or 266)?

1. Induction!! :)
2. Group Theory
3. Graph Theory
4. Counting
5. Sorting
6. Sequences/Summations
7. Proofs
8. Logic
9. Big-O ←

Runtime:

What is big-O?

Worst case running time

Sorting: $O(n^2)$ (quicksort)
$O(n \log n)$ - mergesort

Dfn:

Let f & g be functions from $\mathbb{Z}$ to $\mathbb{R}$.

We say $f(x)$ is $O(g(x))$ if

there exist constants $C > 0$ and $N_0 > 0$ such that for all $n > N_0$,

$$f(n) \leq C \cdot g(n)$$

Ex: Show that x^2+2x+1 is $O(x^2)$.

proof:

For all $n \geq 1$, $n^2 \geq n^2$.

For all $n \geq 1$, $2n^2 \geq 2n$.

For all $n \geq 1$, $n^2 \geq 1$.

So $x^2+2x+1 \leq x^2+2x^2+x^2$
$= 4x^2$

Let $c=4$, then $N_0 = 1$ and

$x^2+2x+1 \leq c \cdot x^2$ ■

Thm: Let $f(x) = \sum_{i=0}^{n} a_i x^i = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ for constants $a_i \in \mathbb{R}$.

Then $f(x) = O(x^n)$.

(Idea: Let $c = |a_n| + |a_{n-1}| + \cdots + |a_1| + |a_0|$)

Other functions

① 2^n versus n^n:

② $\log n$ versus n: