

Math 135: Discrete Mathematics, Spring 2010

Worksheet 9

1. An employee joined a company in 1999 with a starting salary of \$50,000, and every year he gets a raise of \$1,000 plus 5 percent of his salary the previous year. Set up a recurrence relation for the salary of the employee n years after 1000, and then find an explicit formula for his salary.

2. The *Lucas numbers* satisfy the recurrence $L_n = L_{n-1} + L_{n-2}$, where $L_0 = 2$ and $L_1 = 1$.
 - (a) Show that $L_n = f_{n-1} + f_{n+1}$ for $n \geq 2$, where f_n is the n^{th} Fibonacci number.
 - (b) Find an explicit formula for the Lucas numbers.

3. Find *general* solutions of the following recurrences. (Note: this means you don't have to solve for constants!)

(a) $a_n = 5a_{n-1} - 6a_{n-2}$

(b) $b_n = 2b_{n-1} + 3 \cdot (-2)^n$

(c) $c_n = 4c_{n-1} - 4c_{n-2} + (n+1)2^n$

4. Give asymptotic bounds for the following recurrences, using either recursion trees, summations, or Master theorem. Assume reasonable constant base cases of size $O(1)$.

(a) $A(n) = A(n/2) + n$

(b) $B(n) = 2B(n/3) + 4$

(c) $C(n) = 5C(n/5) + 6n$