

Math 135: Discrete Mathematics, Spring 2010

Worksheet 5

- Determine if f is a function from the set of all bit strings (meaning strings where each digit is 0 or 1) to the set of integers for the following possible descriptions of f . If f is a function, also decide if it is 1-1 or onto (or both).
 - $f(S)$ is the position of a 0 bit in S .
 - $f(S)$ is the number of 1 bits in S .
 - $f(S)$ is the smallest integer i such that the i^{th} bit of the string S is 1, and $f(S) = 0$ when S is the empty string
- Prove that if $f : A \rightarrow B$ is onto and $g : B \rightarrow C$ is onto, then $g \circ f$ is onto.
 - Prove that if both f and g are 1-1, then $g \circ f$ is 1-1.

3. Determine if the function $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ is onto and/or 1-1 if

(a) $f(m, n) = m + n$

(b) $f(m, n) = m^2 + n^2$

(c) $f(m, n) = m$

4. Let $f : \mathcal{P}(X) \rightarrow \mathcal{P}(Y)$ be a function such that for all $S, S' \in \mathcal{P}(X)$, $S \subseteq S'$ if and only if $f(S) \subseteq f(S')$. Prove that f is 1-1.

5. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be bijections. Show that $(g \circ f)^{-1} = g^{-1} \circ f^{-1}$.