

Math 135: Discrete Mathematics, Spring 2010

Worksheet 10

1.
 - (a) How many bit strings of length 7 either begin with two 0's or end with three 1's?
 - (b) A palindrome is a string whose reversal is identical to the string. How many bigstrings of length n are there?
 - (c) How many positive integers less than 100 are divisible by 7? How many are divisible by 7 but not 11? How many are divisible by both 7 and 11?
 - (d) A group contains n men and n women. How many ways are there to arrange the people in a row if the men and women must alternate?
 - (e) How many ways are there for 8 men and 5 women to stand in a line so that no two women stand next to each other? (Hint: First position the men and then consider positions for the women.)

2. A computer network consists of 6 computers. Each computer is directly connected to 0 or more of the other computers. Show that there are at least 2 computers in the network connected to the same number of other computers.

3. One hundred tickets, numbered 1 to 100, are sold to 100 different people in a drawing. Four different prizes are awarded, including a grand prize trip to Hawaii. How many ways are there to award prizes if:
- (a) there are no restrictions?
 - (b) the person holding ticket 47 wins the grand prize?
 - (c) the person holding 47 does not win any prize?
 - (d) the people holding tickets 19 and 47 both win a prize?
 - (e) the people holding 19, 47, and 73 all win prizes?
 - (f) none of the people holding 19, 47, and 73 win prizes?
 - (g) the grand prize winner is a person holding 18, 47, 73, or 97?
4. Show that $k \binom{n}{k} = n \binom{n-1}{k-1}$ using a combinatorial proof. (Hint: Consider choosing a committee with a chairperson.)