Math 135 – Sets

Note Title 1/29/2010

Announcements

- HW2 extension – submit by Tuesday @ 5pm
  (in my office!)
- Office hours – Monday from 9:30 – 11
- HW1 is graded
  (get paper + solutions before you leave)
- Worksheet 3 – hand in today!

Sets (Ch 1.6)

5th edition

Dfn: A set is an (unordered) collection of objects.

Ex: $\phi = \{\}$ (the empty set)

$\{1, 3, 5, 7\}$

$\{1, 2, 3, \ldots, 1000\}$

$\{a, b, c, \ldots, z\}$

$\{\phi, \{1\}, \{1, 3\}, \{1, 3, 5\}\}$

Definitions

- A set is said to contain its **elements** (or **members**).
- Two sets are **equal** if & only if they contain the same elements.

Ex: $\{1,3,5,7\} = \{3,7,5,1\}$

$\{1,3,5,7\} = \{1,1,3,3,5,7\}$

(order & multiplicity of members don't matter!)

Examples

Natural Numbers: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

Integers: $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Rational Numbers: $\mathbb{Q} = \{ p/q : p \in \mathbb{Z}, q \in \mathbb{Z}, \text{ and } q \neq 0 \}$

such that

Real Numbers: $\mathbb{R}$ (includes $\mathbb{Q}$ plus π, $\sqrt{2}$, etc...)

Ways to define a set

- List: $S = \{1, 5, 11\}$

  $T = \{1, 2, 3, 4, \ldots\}$

- English: "Let S be the set of squares."
- Form description: $S = \{n^2 : n \in \mathbb{N}\}$
- Property description: $S = \{n \in \mathbb{N} : n \text{ is a perfect square}\}$

Notation:

- $x \in S$ means x is a member of S
- $x \notin S$ means x is <u>not</u> a member of S
- $A \subseteq B$ means that A is a <u>subset</u> of B

  ↳ That is, $\forall x, (x \in A \rightarrow x \in B)$

  <u>Note:</u> $A = B \longleftrightarrow (A \subseteq B$ and $B \subseteq A)$
- $A \subsetneq B$ or $A \subset B$ means A is a <u>proper</u> subset of B

  (so $A \subseteq B$ and $A \neq B$)

Examples:

$$\mathbb{N} \subseteq \mathbb{Z}$$

$$\sqrt{5} \in \mathbb{R}$$

$$\sqrt{2} \notin \mathbb{Q}$$ $\leftarrow$ we proved this!

Lemma: For any set S, $\emptyset \subseteq S$.

proof: Need to show (by definition)

that $\forall x$, if $x \in \emptyset$, then $x \in S$.

$p \longrightarrow q$

p: $x \in \emptyset$

q: $x \in S$

$x \in \emptyset$ is always false.

Remember truth table:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p is always false here

So $p \to q$ is always true $\blacksquare$

Next time: More sets

Set cardinality, Venn diagrams, power sets, & union/intersection