Math 135 - More divide & conquer

Note Title 3/24/2010

Announcements

- HW due Friday
- Midterm in 1 week
- Sample midterm + "cheatsheet" are on website

check this for me

(any suggestions by Monday)

Master theorem:

Let f satisfy $f(n) = a f(\frac{n}{b}) + \Theta(n^k)$,
where $a \geq 1$, b is an integer ≥ 1, and
c and k are real numbers, $c > 0$ & $k \geq 0$.

Then:

$$f(n) = \begin{cases} ① \ O(n^k) & \text{if } a < b^k \\ ② \ O(n^k \log n) & \text{if } a = b^k \\ ③ \ O(n^{\log_b a}) & \text{if } a > b^k \end{cases}$$

How to use:

$$T(n) = 2T\left(\frac{n}{2}\right) + \Theta(n)$$

n^1

Here: $a = 2$

$b = 2$

$k = 1$

So: $a = b^k$

$2 = 2^1$

Case 2, so $T(n) = \Theta(n^1 \log n)$

$= \Theta(n \log n)$

$\frac{n}{b}$

$\frac{3n}{4} = \frac{n}{4/3}$

<u>Ex</u>: $T(n) = T\left(\frac{3n}{4}\right) + n^2$

$a = 1$
$b = 4/3$
$k = 2$

$1 < \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

Case 1 $\Rightarrow$ $T(n) = O(n^2)$

Ex: $T(n) = 3T\left(\frac{n}{2}\right) + n$

$a = 3$

$b = 2$

$k = 1$

So: $3 > 2^1$

case 3 : $T(n) = O\left(n^{\log_2 3}\right)$

$$T(k) = k^{\frac{1}{2}}\, T(k^{\frac{1}{2}}) + k$$

When Master thm doesn't help: use recursion trees!

$$T(n) = \sqrt{n}\, T(\sqrt{n}) + n$$

$$T(\sqrt{n}) = n^{1/4}\, T(n^{1/4}) + n^{1/2}$$

depth d:

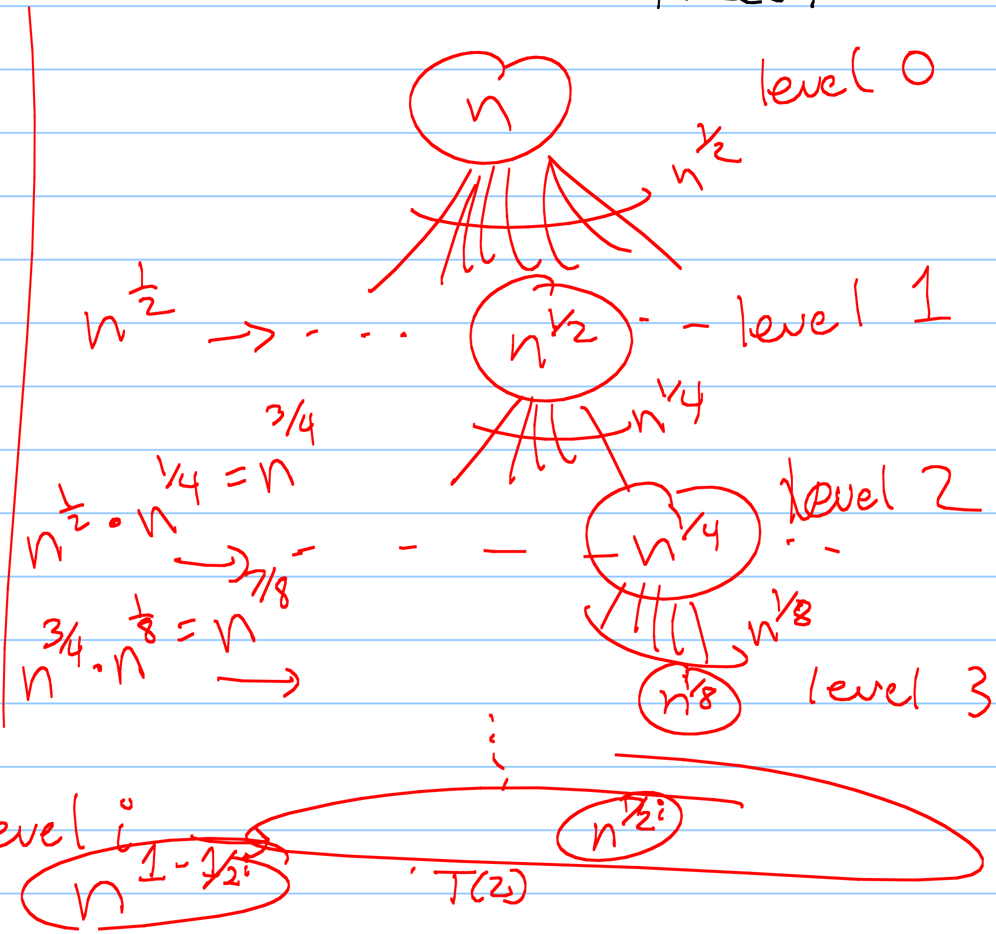
$$n^{\frac{1}{2^d}} = 2$$

$$\log n^{\left(\frac{1}{2^d}\right)} = \log_2 2 = 1$$

$$\frac{1}{2^d} \cdot \log_2 n = 1$$

$$\log_2 n = 2^d$$

$$\log_2(\log_2 n) = d$$



So $T(n) = \sum_{i=0}^{d} (\#\text{ nodes on } i^{th} \text{ level}) * (\text{amount in each node})$

$$= \sum_{i=0}^{\lg\lg n} \left(n^{1-\frac{1}{2^i}}\right) \cdot \left(n^{\frac{1}{2^i}}\right)$$

$$= \sum_{i=0}^{\lg\lg n} n \quad = \Theta(n \lg\lg n)$$

$$T(k) = T\left(\frac{k}{4}\right) + T\left(\frac{k}{2}\right) + k$$

$$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n$$

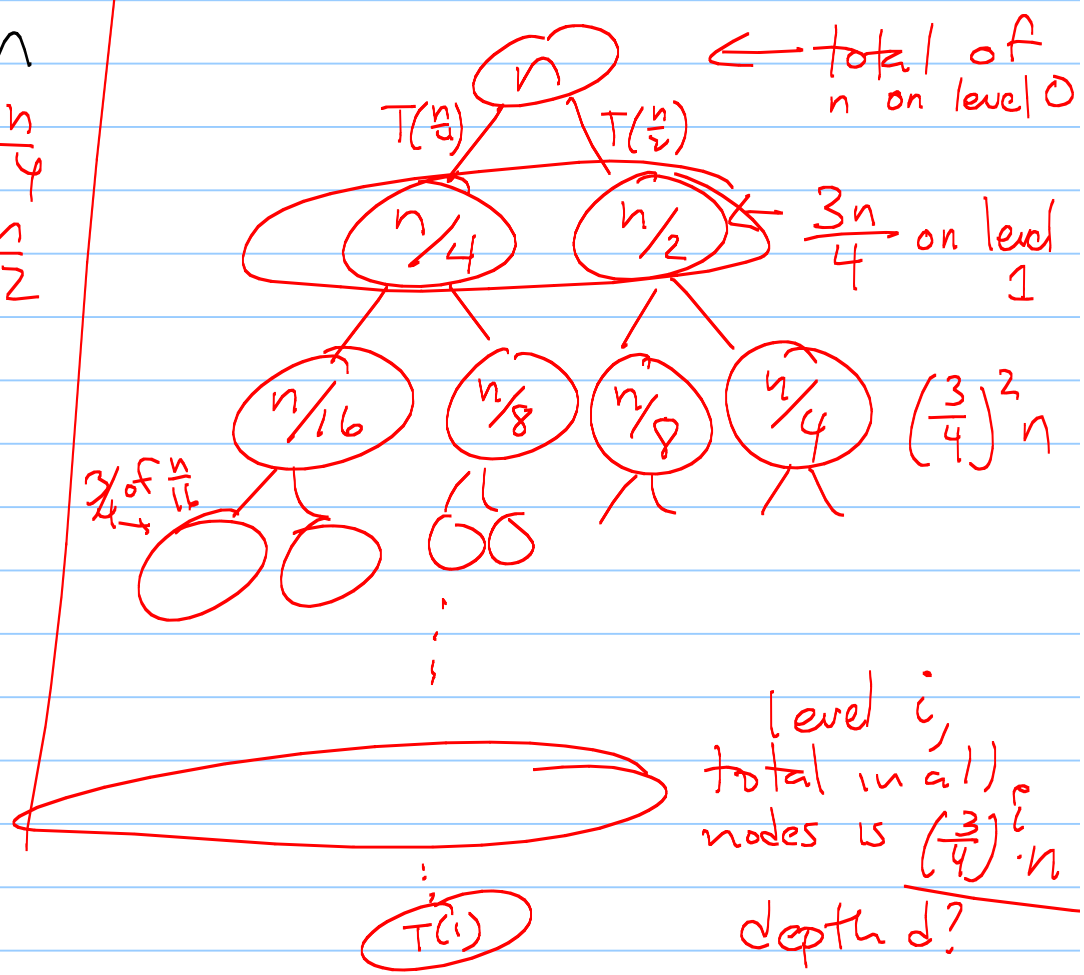
$$T\left(\frac{n}{4}\right) = T\left(\frac{n}{16}\right) + T\left(\frac{n}{8}\right) + \frac{n}{4}$$

$$T\left(\frac{n}{2}\right) = T\left(\frac{n}{8}\right) + T\left(\frac{n}{4}\right) + \frac{n}{2}$$

I divide by 2 or 4 each time

So all leaves will be on levels

$$\log_4 n \le d \le \log_2 n$$



$$\sum_{i=0}^{\log_4 n} \left(\frac{3}{4}\right)^i n \leq T(n) \leq \sum_{i=0}^{\log_2 n} \left(\frac{3}{4}\right)^i \cdot n$$

$$\sum_{i=0}^{\log_2 n} \left(\frac{3}{4}\right)^i \cdot n = n \sum_{i=0}^{\log_2 n} \left(\frac{3}{4}\right)^i$$

$$\leq n \left(\frac{1}{1-3/4}\right) = 4n$$

so $T(n) = O(n)$

$$n \leq \sum_{i=0}^{\log_4 n} \left(\frac{3}{4}\right)^i n \leq T(n)$$

so $T(n) = \Omega(n)$

$$T(n) = \Theta(n)$$