Math 135 - Functions

Note Title 2/8/2010

Announcements

- HW due Friday
- No office hours on Thursday 1-2pm — instead, will be in from 9-10 am
  - (Also, don't forget usual office hours on Wed. morning)
- Midterm 1 - next Wednesday in class!
  - Sample midterm will be up by Friday, & review session in class on Monday.

A note about homework:

"Prove or disprove".

Prove — requires a proof for full credit!

Disprove — requires a counterexample

(or a proof that it does not hold,
but usually counterexample is easier)

Functions

Recap: Functions map elements from one set to another. $f: A \to B$

Defns: domain:

codomain versus range:

onto:

1-1:

bijection:

inverse

Thm: Functions $f: A \to B$ and $g: B \to A$ are inverses of each other $\iff$ $f \circ g = i_B$ and $g \circ f = i_A$

proof: Two directions!

"$\Leftarrow$": Suppose $f \circ g = i_B$ and $g \circ f = i_A$.

Need to show f & g are inverses of each other, so $\forall a \in A$ and $\forall b \in B$, $f(a) = b \iff g(b) = a$.

$\Rightarrow$: Spps $f(a) = b$. Apply g to both sides.

$$g(f(a)) = g(b)$$

$$\| $$

$$(g \circ f)(a) = i_A(a) = a.$$

$\Leftarrow$: Spps $g(b) = a$. Apply f: $f(g(b)) = f(a)$

so $g(b) = a$

& $f(g(b)) = (f \circ g)(b) = i_B(b) = b$. $\Rightarrow f(a) = b$.

"$\Rightarrow$": Suppose f & g are inverses of each other.
Need to show $f \circ g = i_B$ & $g \circ f = i_A$.

① Show $g \circ f = i_A$.
Take any $a \in A$, & let $b = f(a)$.

Apply g to both sides:

$$g(b) = g(f(a))$$

Know $g(b) = a$, since $f(a) = b$ & f & g are inverses.

So $g(f(a)) = (g \circ f)(a) = a \quad \forall a \in A$.

So $g \circ f$ is the identity (by defn).

② Similar argument — exercise

□

Thm: Let A & B be finite sets, with $f: A \to B$.

a) If f is 1-1, then $|A| \leq |B|$.

b) If f is onto, then $|A| \geq |B|$.

proof of b:

exercise

Cor: If $f: A \to B$ is a bijection, then $|A| = |B|$.

Powerful Technique!

Ex: Poker - played with cards in 4 suites, $\{\heartsuit, \clubsuit, \spadesuit, \diamondsuit\}$

and 5 cards in hand.

Show that some suite must appear twice.

Let f be a function $f: A \to B$.

$A = \{\text{cards in hand}\}$

$B = \{\text{suites}\}$

more next time...