Math 135 - lecture 3

Note Title 1/15/2010

Announcements

- Turn in HW0 (now)
- HW1 is out
- In class group work/quiz today
- No class Monday

Proofs:

- A theorem (or lemma, or proposition) is a statement that can be rigorously shown to be true.
- The sequence of statements giving that argument is called a proof.

Direct proofs:

Think about statement $p \to q$.
When is it false?

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

To show $p \to q$ is true, need to show that if p is true, q cannot be false.

Ex: If n is an odd integer, then n^2 is an odd integer.

pf: (Assume p is true, then show q cannot be false.)

Assume n is an odd integer.

That means we can write n as $n = 2k+1$ for some $\underbrace{k \in \mathbb{Z}}_{k \text{ is an integer}}$.

Then: $n^2 = (2k+1)^2$
$= (2k+1)(2k+1)$
$= 4k^2 + 4k + 1$
$= 2\underbrace{(2k^2+2k)}_{\text{this is an integer}} + 1$ $\Rightarrow n^2$ is odd.

$\blacksquare$

Indirect Proofs:

Recall: $p \to q$ is logically equivalent to $\neg q \to \neg p$.

(What is $\neg q \to \neg p$ called?) Contrapositive

Since they are equivalent, showing $p \to q$ is true can instead be accomplished by showing $\neg q \to \neg p$.

Ex: If $3n+2$ is odd, then n is odd.

$\neg q \to \neg p$

Pf: (Assume $\neg q$ is true & show $\neg p$ must also be true.)

(indirect proof)

Assume n is even.

That means $n = 2k$ for some $k \in \mathbb{Z}$.

Then: $$3n+2 = 3(2k)+2 = 6k+2 = 2\underbrace{(3k+1)}_{\text{is an integer}}$$

So $3n+2$ is even $\blacksquare$