

# Math 135 - Lecture 2

Note Title

1/13/2010

## Announcements

- HW due Friday!
- New HW will come out Friday & will be due next week.
- Office Hours: Wed. 9:30 - 11  
Thurs. 1 - 2

Last time: Logic

- propositions & their negations

- truth tables

- logical equivalence

- contrapositive, converse, inverse

A more applied exercise:

truth tellers & liars

Suppose we meet two people, Alice and Bob.

Alice says: "Exactly one of us is lying."

Bob says: "At least one of us is telling the truth."

How do you tell who is lying & who is honest?

Let  $p = \text{"Alice is truthful"}$   
 $q = \text{"Bob is truthful"}$

	<u>Statement 1</u> Exactly 1 is lying	<u>Statement 2</u> At least 1 is truthful
$p$	T	F
$q$	T	T
T	F	T
F	T	T

Which row is consistent?

Another one:

Alice says: "Exactly one of us is telling  
the truth."

Bob says: "We are all lying."

Cindy says: "The other two are lying."

Same principle, bigger table!



Predicates:  $P(x)$

propositions that depend on some variable

Ex:  $x > 3 \rightarrow P(x)$

$x = y + 3 \rightarrow Q(x, y)$

" $x$  is in discrete math"  $\rightarrow R(x)$

" $x$  is a SLU student"  $\rightarrow S(x)$

Note:

- Can combine these
- truth value depends on variable

$$P(5) = \text{true}$$

$$Q(1, 11) = \text{false}$$

$$R(x) \wedge S(x)$$

(true if  $x$  is a student in this class)

## Quantifiers

$\forall x P(x)$ : For all  $x$  (in universe),  $P(x)$  is true.

universal quantifier

Ex: Let  $P(x) = "x+1 > x"$ , and  $Q(x) = "x < 2"$

What are the truth value of :

$\forall x \in \mathbb{R}, P(x)$ : For all reals  $x$ ,  $x+1 > x$ .

True

$\forall x \in \mathbb{R}, Q(x)$ : For all reals  $x$ ,  $x < 2$ .

False

## Quantifiers

$\exists x P(x)$ : There exists  $x$  (in universe) such that  $P(x)$  is true.

### Existential Quantifier

Ex: Let  $P(x) = "x > 3"$  and  $Q(x) = "x = x + 1"$

$\exists x P(x)$ : There is a real #  $x$  such that  $x > 3$ .  
↳ True

$\exists x Q(x)$ : There is a real #  $x$  s.t.  $x = x + 1$ .  
↳ False

These can get more complicated!

$$\exists x (P(x) \wedge Q(x)) \vee \forall x R(x)$$

Which quantifier holds where?

There is an  $x$  such that  $P(x)$  is true and  
 $Q(x)$  is true  
OR for all  $x$ ,  $R(x)$  is true.

## Negations

How should we negate quantifiers?

Consider the following:

$P(x)$  = "x has taken college algebra."

$\neg P(x)$  = "x has not taken college algebra."

So  $\forall x P(x)$  is "Every student has taken"

college algebra."

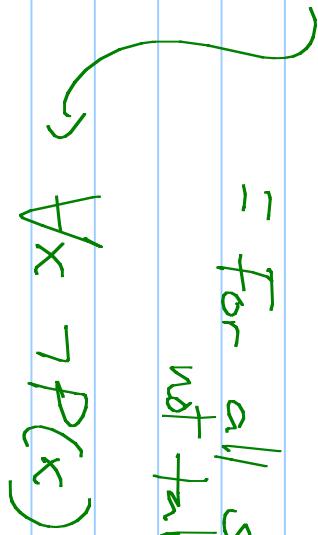
What is  $\neg (\forall x P(x))$ ?

$\rightarrow \exists x \neg P(x)$

↳ There is a student who has not taken college algebra.

What about  $\exists x P(x) =$   
"There is a student who has taken  
college algebra."

$\neg (\exists x P(x)) =$  There is no student who has  
taken college algebra.  
 $=$  For all students  $x$ ,  $x$  has  
not taken college algebra.



$$\forall x \neg P(x)$$

Nested quantifiers:

(assume universe is  $\mathbb{R}$ )

$\forall x \exists y (x+y=0)$  : true

translate: For all  $x$  there exist a  $y$  s.t.  $x+y=0$ .

What about:

$\exists y \forall x (x+y=0)$  : false

translate: There is a  $y$  s.t. for every  $x$ ,  $x+y=0$ .

Another one: If our universe is  $\mathbb{R}$ ,

$$\forall x \forall y ((x > 0) \wedge (y < 0)) \rightarrow (xy < 0)$$

for all  $x$  and all  $y$ ,

if  $x > 0$  and  $y < 0$ ,

then  $xy < 0$ .

## Negating implications

What is  $\neg(p \rightarrow q)$ ?  $\neg(p \rightarrow q) \equiv p \wedge \neg q$

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg q$	$p \wedge \neg q$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	T	F	F
F	F	T	F	F	F

Ex: Write negation of: "If Bob has an 8am class today, then it is Tuesday."

So:

What is  $\neg(\forall x P(x) \rightarrow Q(x))$

$\exists x (P(x) \wedge \neg Q(x))$

Ex: ( $\forall$  real numbers  $x > 0$ , if  $x^2 = 1$ , then  $x^3 = 1$ .)  
Write negation:

There is a real #  $x > 0$  s.t.  
 $x^2 = 1$  and  $x^3 \neq 1$ .