Math 135 - lecture 2

Note Title 1/13/2010

Announcements

- HW due Friday!
- New HW will come out Friday & will be due next week.
- Office Hours: Wed. 9:30-11
  Thurs. 1-2

<u>Last time</u>: Logic

- propositions & their negations
- truth tables
- logical equivalence
- contrapositive, converse, inverse

A more applied exercise:
truth tellers & liars

Suppose we meet two people, Alice and Bob.

Alice says: "Exactly one of us is lying."

Bob says: "At least one of us is telling the truth."

How do you tell who is lying & who is honest?

Let p = "Alice is truthful"
q = "Bob is truthful"

p	q	Statement 1 Exactly 1 is lying	Statement 2 At least 1 is truthful
T	T	F	T
T	F	T	T
F	T	T	T

Which row is consistent?

Another one:

Alice says: "Exactly one of us is telling the truth."

Bob says: "We are all lying."

Cindy says: "The other two are lying."

Same principle, bigger table!

p	q	r	Exactly 1 truthful	All lying	Other 2 lying
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	F	F
T	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
F	T	T	F	F	F
F	T	F	T	F	F
F	F	T	T	F	T
F	F	F	F	T	T

Predicates : $P(x)$

propositions that depend on some variable

Ex: $x > 3 \longrightarrow P(x)$

$x = y + 3 \longrightarrow Q(x, y)$

"x is in discrete math" $\longrightarrow R(x)$

"x is a SLU student" $\longrightarrow S(x)$

Note:
- Can combine these
- Truth value depends on variable

$P(5) = $ true

$Q(1, 11) = $ false

$R(x) \wedge S(x)$

(true if x is a student in this class)

Quantifiers

$\underbrace{\forall x}_{\text{universal quantifier}} P(x)$: For all x (in universe), $P(x)$ is true.

Ex: Let $P(x) =$ "$x+1 > x$", and $Q(x) =$ "$x < 2$"

What are the truth value of :

$\forall x \in \mathbb{R}, P(x)$: For all reals x, $x+1 > x$.

True

$\forall x \in \mathbb{R}, Q(x)$: For all reals x, $x < 2$.

False

Quantifiers

$\exists x\, P(x)$: There exists x (in universe) such that $P(x)$ is true.

Existential Quantifier

Ex: Let $P(x) =$ "$x > 3$" and $Q(x) =$ "$x = x+1$"

$\exists x\, P(x)$: There is a real # x such that $x > 3$.
↳ True

$\exists x\, Q(x)$: There is a real # x s.t. $x = x+1$.
↳ False

These can get more complicated:

$$\exists x\,(P(x) \wedge Q(x)) \vee \forall x\, R(x)$$

Which quantifier holds where?

There is an x such that $P(x)$ is true and $Q(x)$ is true

OR for all x, $R(x)$ is true.

Negations

How should we negate quantifiers?

Consider the following:

$P(x) =$ "x has taken college algebra."

$\neg P(x) =$ "x has not taken college algebra."

So $\forall x\, P(x)$ is "Every student has taken college algebra."

What is $\neg(\forall x\, P(x))$? → $\exists x\, \neg P(x)$

↳ There is a student who has not taken college algebra.

What about $\exists x\, P(x)$ =
"There is a student who has taken college algebra."

$\neg(\exists x\, P(x))$ = There is no student who has taken college algebra.

= For all students x, x has not taken college algebra.

$\forall x\, \neg P(x)$

Nested quantifiers:

(assume universe is $\mathbb{R}$)

$\forall x \, \exists y \, (x+y=0)$: true

translate: For all x there exist a y s.t. $x+y=0$.

What about:

$\exists y \, \forall x \, (x+y=0)$: false

translate: There is a y s.t. for every x, $x+y=0$.

Another one: If our universe is $\mathbb{R}$,

$$\forall x \, \forall y \, \Big(\big((x>0) \wedge (y<0) \big) \rightarrow (xy<0) \Big)$$

For all x and all y,

if $x>0$ and $y<0$,

then $xy<0$.

Negating implications

What is $\neg(p \to q)$? $\quad \neg(p \to q) \equiv p \wedge \neg q$

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$p \wedge \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

Ex: Write negation of: "If Bob has an 8am class today, then it is Tuesday."

So:

What is $\neg(\forall x\, P(x) \to Q(x))$

$\exists x\, (P(x) \wedge \neg Q(x))$

Ex: $\neg$($\forall$ real numbers $x > 0$, if $x^2 = 1$, then $x^3 = 1$.)

Write negation:

There is a real # $x > 0$ s.t. $x^2 = 1$ and $x^3 \neq 1$.