Math 135 - Discrete Math

Note Title 1/11/2010

- Syllabus is posted
- HW0 - due this Friday

Logic

Dfn: A proposition is a declarative statement which is either true or false, but not both.

Ex: The sky is blue.

Money is worthless.

Ambiguous: I like ice cream.

The sky is pretty.

Negation

$\neg p$

Let p be a proposition.
The negation of p, written $\neg p$, is the statement:
"It is not the case that p"

Ex: p = "The sky is yellow"

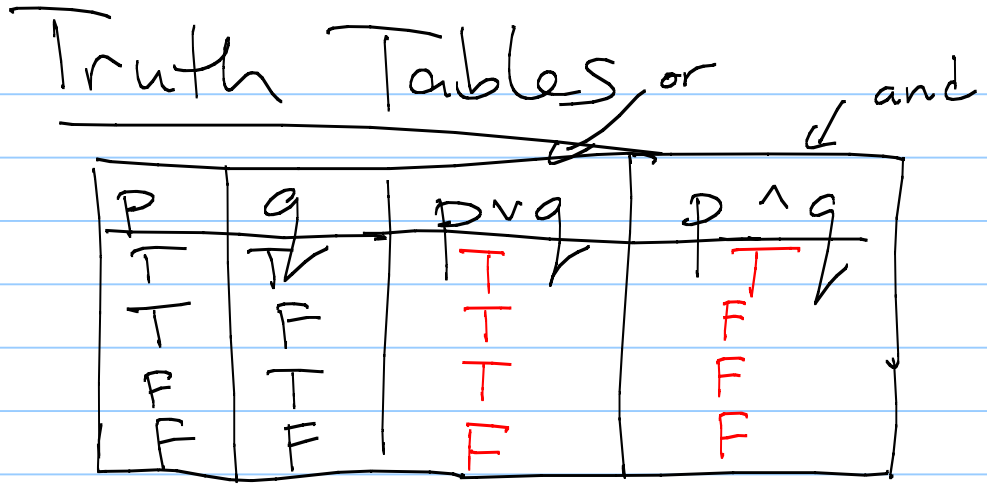
$\neg p$ = "The sky is not yellow"

Conjunction and Disjunction
(ie "and" and "or")

Conjunction: "p and q", written $p \wedge q$, is true exactly when both p & q are true & is false otherwise

Disjunction: "p or q", written $p \vee q$, is true if either p or q is true, or if both are true

Truth Tables



Note: Sometimes write 0 for F
1 for T

Exclusive Or: $p \oplus q$

Is true is either p or q is true, but false if both are true or if both are false.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implications: $p \rightarrow q$

"if p, then q"

"p implies q"

"q if p"

Ex: "If I am elected, then I will lower taxes."

When is it true?

Dfn: $p \to q$ is false when p is true and q is false
(and is true otherwise!)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

} call this "vacuously true"

Examples to ponder:

"If it is sunny, then we will go to the beach."

"If today is Friday, then $2+5=7$."

Converse, Inverse, & Contrapositive

The converse of $p \to q$ is the statement $q \to p$.

The inverse is $\neg p \to \neg q$.

The contrapositive is $\neg q \to \neg p$.

Exercise: Draw the truth tables.

Logical Equivalence

Propositions that have the same truth values are called logically equivalent.

(written $p \equiv q$).

Example?

$$(p \to q) \equiv (\neg q \to \neg p)$$

Example: $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Why?

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

De Morgan's Law