

Math 135 - Infinite Sets

Note Title

2/8/2010

Announcements

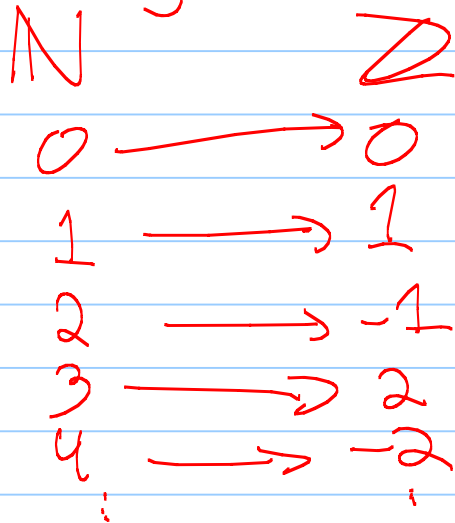
- HW due Friday
- Office hours tomorrow 9-10am
- Midterm next Wed.

Infinite Sets (Ch. 2.4, end of section)

Dfn: Two sets A and B have the same cardinality if and only if there is a bijection from A to B . \cup

Thm: \mathbb{N} and \mathbb{Z} have same cardinality.

pf: dovetailing



} this is
a bijection

← rational #s

Thm: \mathbb{N} and \mathbb{Q} have same cardinality.

pf (picture)
numerator

denom.	1	2	3	4	5	...
1	$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{1}$	$\frac{4}{1}$...	
2	$\frac{1}{2}$	$\frac{2}{2}$	$\frac{3}{2}$...		
3	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$...		
4		
5	
6
...

$\frac{p}{q}$ ✓

Are there sets which are "bigger" than \mathbb{N} ?

Dfn: A set A is countable if there is a bijection $f: \mathbb{N} \rightarrow A$ (or if A is finite).

→ Q: are there uncountably infinite sets?

\mathbb{R}

$\mathcal{P}(\mathbb{N})$

Thm: \mathbb{R} is not countable.

Actually, we'll show $(0,1) \subseteq \mathbb{R}$ is not countable.

Called Cantor diagonalization argument:

pf: Look at decimal representations of numbers $\in (0,1)$.
Proof by contradiction!

Suppose there is a bijection $f: \mathbb{N} \rightarrow \mathbb{R}$,

Consider that function as giving a
list of all decimal #s between
 0 & 1 .

1 st	•	X_{11}	X_{12}	X_{13}	X_{14}	...
2 nd	•	X_{21}	X_{22}	X_{23}	X_{24}	...
3 rd	•	X_{31}	X_{32}	X_{33}	X_{34}	...
	•	X_{41}	X_{42}	X_{43}	X_{44}	...
	•					...

•	0	1	5	3	...
•	3	2	5	9	...
•	6	1	2	4	...
•	2	4	3	9	...

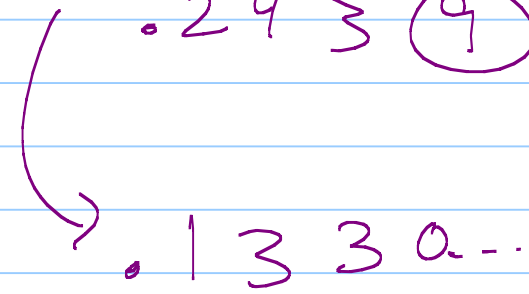
Construct another decimal #:

$d = .d_1 d_2 d_3 d_4 d_5 \dots$

let $d_i = X_{ii} + 1$

so $d_i \neq X_{ii}$ for all i

So d is different from every row of my listing!



But why do we care??

Well, we care about computable things.

How many computer programs are there?

So a computer program can be "just" a number!

How many functions from $\mathbb{N} \rightarrow \{0, 1\}$ are there?

x	0	1	2	3	4	...
$f(x)$						

these look like fractions b/t 0 and 1

\Rightarrow there are a lot of uncomputable functions!