Math 135 - Sets - Ch 2

Note Title 2/3/2010

Announcements

- Homework - out tomorrow/Friday,
  due next Friday (?)
- No afternoon office hours next Thursday -
  will reschedule to Thursday morning.

Last time: De Morgan's Law

Prove $\overline{A \cap B} = \overline{A} \cup \overline{B}$:

Stopped here: 2 Show $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

Consider $x \in \overline{A} \cup \overline{B}$.

So $x \in \overline{A}$ or $x \in \overline{B}$, so $[(x \notin A) \text{ or } (x \notin B)]$

So $\neg(x \in A)$ or $\neg(x \in B)$ which by logic version of D.M.

$\Rightarrow \neg[x \in A \text{ and } x \in B]$, so $\neg(x \in A \cap B)$

$\Rightarrow x \notin A \cap B \Rightarrow x \in \overline{A \cap B}$ ∎

Notation:

We will write

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$$

Similarly,

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$

Tuples

In sets, order doesn't matter: $\{1,2\} = \{2,1\}$

(But sometimes, order **does** matter!)

A **tuple** is an ordered list of objects

Ex:
- $(4,2,8)$
- $()$
- $(\emptyset, \{2\}, \{3,8\})$
- $(1,2) \neq (2,1)$

A tuple with n entries is an **n-tuple**.
(If $n=2$, called **ordered pair**.)

Cartesian Product

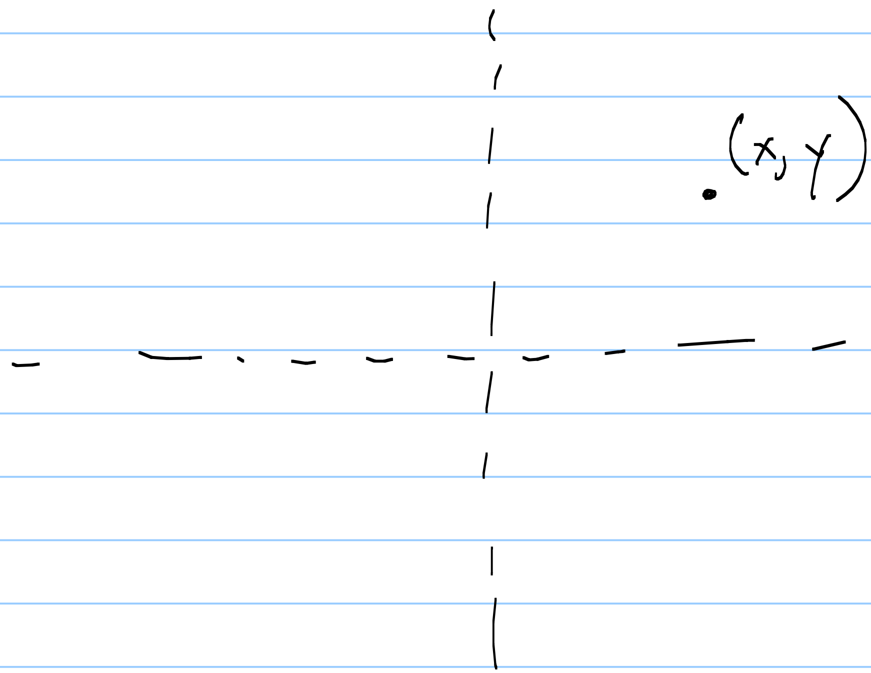
Dfn: Given sets A & B, the product of A & B (written $A \times B$) is the set of 2-tuples where first element is from A and second element is from B.

$$A \times B = \{ (a,b) \mid a \in A \text{ and } b \in B \}$$

Ex: $A = \{a, b, c\}$ $B = \{1, 2\}$

$$A \times B = \{ (a,1), (b,1), (c,1), (a,2), (b,2), (c,2) \}$$

Another: $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$



$\neq (y, z)$

With more than 2 sets, have:

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, \dots, a_n) \mid \forall i,\ a_i \in A_i\}$$

Notation: $A^n = \underbrace{A \times A \times \cdots \times A}_{n \text{ times}}$

(Hence, $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, & $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$)

Caution! $(A \times B) \times C \neq A \times B \times C$

Typical element in $(A \times B) \times C$ is $((a, b), c)$

But in $A \times B \times C$, (a, b, c)

Another: What is $\emptyset \times \{a, b\}$?

$= \emptyset$

Russell's Paradox

- Sets are basic mathematical objects — but be careful of contradictions!

Ex: Let A be the set of sets which do not contain themselves:

$$A = \{ S \mid S \notin S \}$$

Question: Is $A \in A$?

Now every element in A is a set which does not contain itself. So $A \in A$ is impossible.

But then $A \notin A$, so A is a set which does not contain itself.
That means $A \in A$ by definition!

Either way, we have a contradiction!

Solution: to keep mathematics whole, we declare that A is not a set!

Most set theory begins w/ assumption that $\emptyset$ + $\mathbb{N}$ are sets, + provides rules for building up.

Ex rule: If S is a set, $P(S)$ is a set.

These rules don't allow us to construct A.

In practice, we won't worry too much - our sets will be legal.

(See Naive Set Theory by Halmos if you're curious, or go take a logic course. :))

Functions

Let A & B be sets. A function from A to B is an assignment of exactly one element of B to each element of A.

We write $f(a) = b$ where $a \in A$, $b \in B$.

Often write $f: A \to B$ to denote a function f.

A is the domain of f, & B is the co-domain.