Math 135 - Last day of counting!

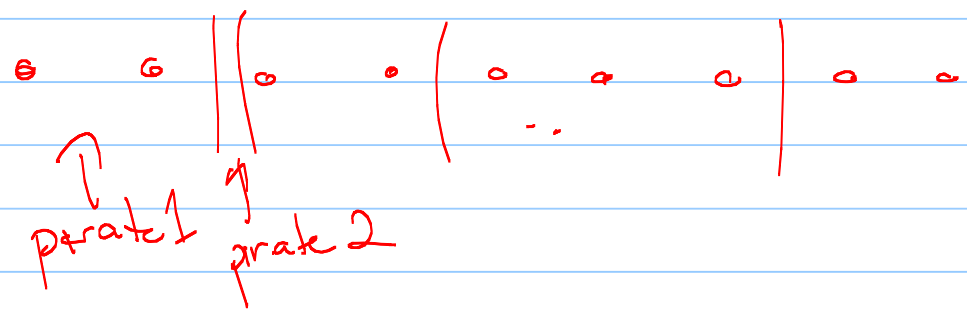
Note Title 4/16/2010

Announcements

- HW due
- Next HW posted, due in 1 week (next Friday)
- Final is May 7, 3-5pm

Last time:

n pirates + r coins



$$\binom{r+n-1}{r} = \binom{r+n-1}{n-1}$$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

Permutations with indistinguishable objects

How many different strings can we make by reordering letters in the word "SUCCESS"?

$$\binom{7}{2}\binom{5}{3}\binom{2}{1}\binom{1}{1}$$

C S C _ S S _

$$\boxed{\binom{7}{3}\cdot\binom{4}{2}\binom{2}{1}\binom{1}{1}} = \frac{7!}{3!\,\cancel{4!}}\cdot\frac{\cancel{4!}}{2!\,\cancel{2!}}\cdot\frac{\cancel{2!}}{1!\,1!}\cdot 1 = \frac{7!}{3!\,2!}$$

place S's, C's, U, E

Thm: The number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 of type 2, ..., & n_k of type k, is

$$\frac{n!}{n_1! \cdot n_2! \cdot \cdots \cdot n_k!}$$

So, in SUCCESS, 3 indistinguishable S's + 2 indistinguishable C's,

$$\text{So} = \frac{7!}{3!\,2!}$$

Putting objects into boxes

① Distinguishable boxes + distinguishable objects

Ex: How many ways are there to deal 5 card poker hands to 4 players?

$$\binom{52}{5}\cdot\binom{47}{5}\cdot\binom{42}{5}\cdot\binom{37}{5} = \frac{52!}{5!\cdot\cancel{47!}}\cdot\frac{\cancel{47!}}{5!\cdot\cancel{42!}}\cdot\frac{\cancel{42!}}{5!\cdot\cancel{37!}}\cdot\frac{\cancel{37!}}{5!\cdot 32!}$$

$$= \frac{52!}{5!\,5!\,5!\,5!\,32!}$$

$x^0 = 1$ $\boxed{0! = 1}$

S_2:

1. Distinguishable boxes + distinguishable objects

Thm The # of ways to distribute n distinguishable objects into k distinguishable boxes so that n_i go into box i (for $i = 1, \dots, k$) is

$$\frac{n!}{n_1!\, n_2! \cdots n_k!}$$

In poker example: I dealt 5 cards per player, & had a "box" for the rest of the deck.

$$\Rightarrow \frac{52!}{5!\,5!\,5!\,5!\,32!}$$

② Indistinguishable objects & distinguishable boxes

Ex: How many ways are there to put 10 identical balls into 8 distinguishable bins?

o || o o || o o o | o | o | o o

Coins & pirates!

$10 = r, \quad 8 = n$

$$\binom{17}{7} = \binom{17}{10}$$

3. Distinguishable objects into indistinguishable boxes
4. Indistinguishable objects & indistinguishable boxes

Nontrivial :)

(see end of 5.5 if you are curious)