# Math 135 - More Permutations + Combinations

Note Title 4/14/2010

## Announcements

- Office hours tomorrow changed:
  will be 10(ish) to noon

- HW due Friday

- Next HW will be due 1 week from Friday
  (a bit more counting - should be a little shorter)

# Worksheet Recap:

(1b) How many palindromes (bitstrings) of length $n$ are there?

(assume $n$ is even)

$2^{\frac{n}{2}}$ — — — — X X X X

$2 \cdot 2 \cdot 2 \cdot 2$

$2^{\lceil \frac{n}{2} \rceil}$

for $n$ odd:

$2^{\frac{n+1}{2}}$ — — — X X

$2 \quad 2 \quad 2$

(12) A group contains $n$ men & $n$ women. How many ways are there to arrange them in a row if they must alternate?

arrange the men

$\underline{(n)}\ \underline{(n-1)}\ \underline{(n-2)}\ \underline{\quad}\ \underline{\quad}\ \cdots\ \underline{1} = n!$

$n$ ways; $n-1$; $n-2$; $\cdots$; $P(n,n) = n!$

$= (n!)(n!)(2)$

$(n!)$ arrange men; $(n!)$ arrange women; $(2)$ lead with woman or a man

② A network has 6 computers, each connected to 0 or more other computers. Show that at least 2 in the network connected to the same # of other computers.

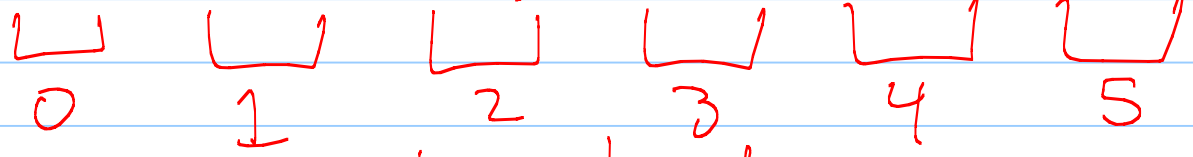
pigeonhole!

objects: computers

boxes: # of other computers it connects to

only 5 of them can be occupied

So 2 are in the same box!




if box 0 is not empty, box 5 is empty

(& vice versa)

(3) 100 tickets sold to 100 different people.
4 prizes (grand prize is trip to Hawaii)
How many ways to award if:

$\binom{100}{4} \cdot 4!$ – no restrictions? $100 \cdot 99 \cdot 98 \cdot 97 = P(100, 4)$

$\binom{99}{3} \cdot 3!$ – person holding 47 wins the grand prize? $P(99, 3)$

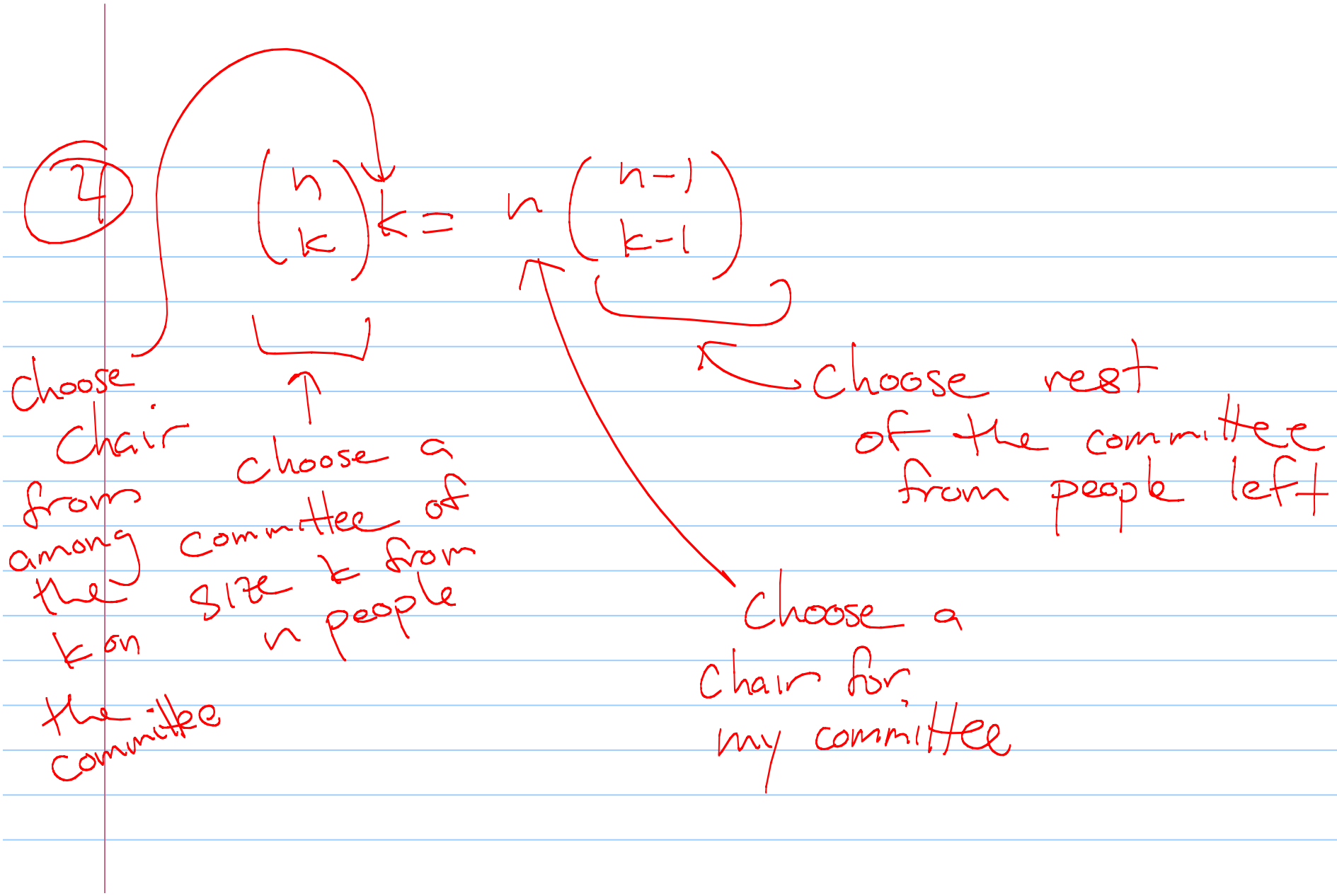
$\binom{99}{4} \cdot 4!$ – person with 47 doesn't win? $P(99, 4)$

– person with 47 wins some prize?

$4 \cdot P(99, 3) = P(99, 3) + P(99, 3) + P(99, 3) + P(99, 3)$

– both 19 & 47 win a prize?

$4 \cdot 3 \cdot P(98, 2)$


4
$\binom{n}{k} k = n\binom{n-1}{k-1}$
choose chair from among the k on the committee
choose a committee of size k from n people
choose rest of the committee from people left
Choose a chair for my committee

# Permutations with repetition (5.5)

How many strings of length $r$ can be formed from English alphabet?

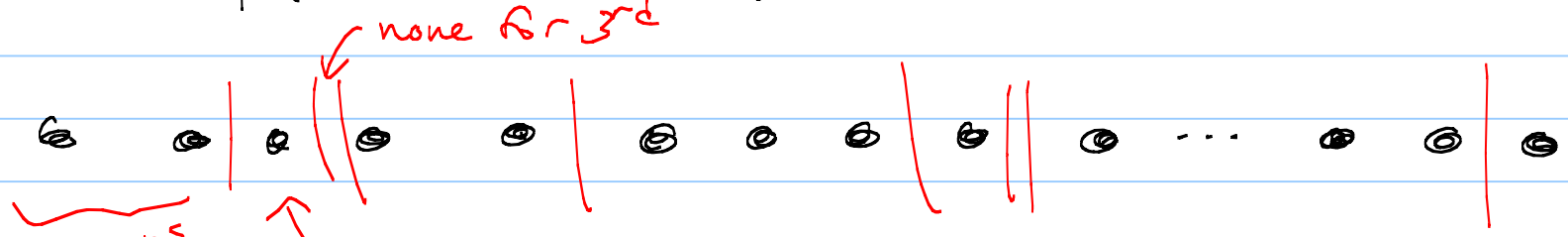
$$\underbrace{26 \cdot 26 \cdot 26 \cdots}_{r} = 26^r$$

[ Note - not $P(26, r) = 26 \cdot 25 \cdot 24 \cdots (26-r+1)$ ]

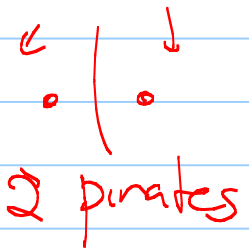
# Combinations

How many ways are there to distribute r identical gold coins among n pirates?

Trick: Place coins in a row:




How can we divide?




for n pirates, need n-1 dividers

In total, have $\underbrace{r}_{\#\text{ coins}} + \underbrace{(n-1)}_{\#\text{ bars}}$

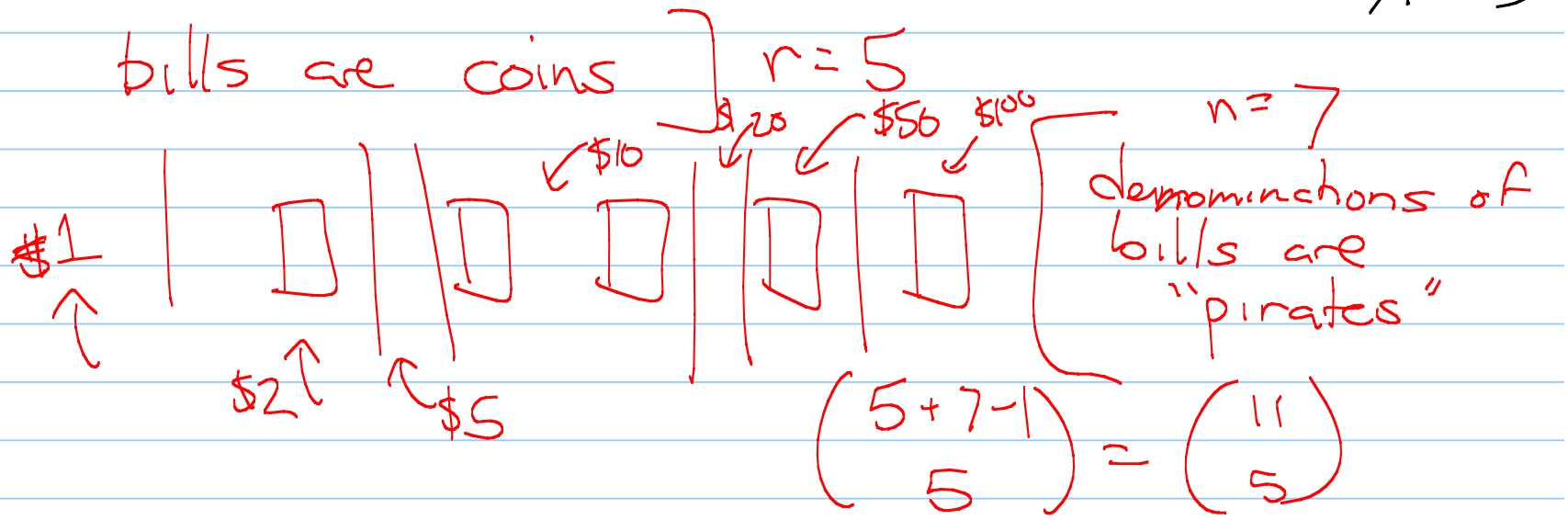
Need to choose $r$ spaces for the coins; rest become bars

$\underbrace{\text{- - - - - - - - - -}}_{r+n-1 \text{ positions}}$

Ans: $\boxed{\binom{r+n-1}{r}} = \binom{r+n-1}{n-1}$

Q: How many ways are there to select 5 bills from a cash drawer containing $1 bills, $2 bills, $5, $10, $20, $50, and $100 bills?

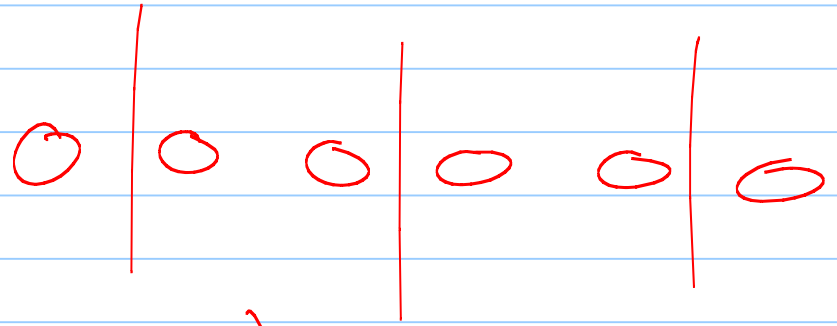
(Assume bills of same type are indistinguishable, and that we have at least 5 of each type.)



Q: Suppose a cookie shop has 4 different kinds of cookies. How many different ways to choose 6 cookies?

$n = 4$

$r = 6$



$$\binom{6+4-1}{6} = \binom{9}{6}$$

Q: How many non-negative integer solutions are there to:

$$\underbrace{x_1 + x_2 + x_3 + x_4 + x_5}_{n=5} = 100?$$

$r = 100$

$$\binom{r+n-1}{n-1}\binom{104}{4} = \binom{100+5-1}{100} = \binom{104}{100} = \binom{r+n-1}{r}$$