Math 135- More Counting

Note Title 4/7/2010

Announcements

- Midterm 2: average 69% (or 45.32)

  Extra credit opportunity:

  redo lowest 2 problems & resubmit on Friday

- Next HW out Friday, due in 1 week

Last time:

1. Rule of Sum

2. Rule of Product

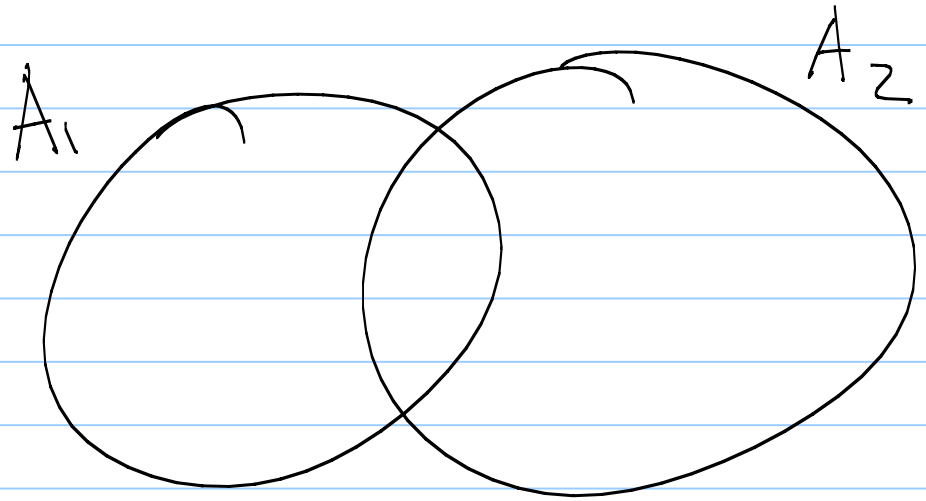
Example

Suppose you need to make a password.
- 6 to 8 characters long
- Uppercase letters or numbers
- At least 1 digit.

How many are possible?

Principle of Inclusion/Exclusion

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$



Ex: How many bit strings of length n either start with a 1 or end with 00?

Sec. 5.2 : The Pigeonhole Principle

Thm: If k is a positive integer and $k+1$ objects are placed into k boxes, then some box contains 2 or more objects.

Proof:

Examples:

- A function from a set with $k+1$ elements to a set with k elements is not 1-1.

- In any group of 367 people, 2 have the same birthday.

- In any group of 27 words, some 2 start with the same letter.

Better: Show that for every integer n, there is a multiple of n that is written with only 0's & 1's (in decimal).

Generalized pigeonhole principle

If N objects are placed into k boxes, then there is a box containing at least $\lceil N/k \rceil$ objects.

pf:

Example: Among 100 people, how many must be born in same month?

Ex: How many cards must we select from a standard 52 card deck in order to be sure that 3 are of the same suite?

Ex: During a month with 30 days, a baseball team plays at least 1 game a day, but no more than 45 total.
Show that there is a period of consecutive days where the team plays exactly 14 games.

Ex: Among any $n+1$ positive integers which are $\leq 2n$, one integer must divide another.