Math 135 - Big Ω & Θ

Note Title 2/22/2010

- Midterm:

  Average: 58.6 (~78%)

  Max: 75, Min: (in 30s)

- New HW will be posted tonight

Big-Omega

Dfn: Let f & g be functions from $\mathbb{R} \to \mathbb{R}$ (or $\mathbb{Z} \to \mathbb{R}$)

We say $f(x)$ is $\Omega(g(x))$ if $\exists$ positive constants C, k such that

$$|f(x)| \geq C \cdot |g(x)| \quad \text{when } x > k.$$

(Read - f is big-omega of g).

already know $\sum_{i=1}^{n} i = O(n^2)$

Ex: $\sum_{i=1}^{n} i = \Omega(n^2)$.

$\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + \frac{n}{2} + \left(\frac{n}{2}+1\right) + \cdots + n$

$\geq \frac{n}{2} + \left(\frac{n}{2}+1\right) + \left(\frac{n}{2}+2\right) + \cdots + n$

$\geq \underbrace{\frac{n}{2} + \frac{n}{2} + \frac{n}{2} + \cdots + \frac{n}{2}}_{n/2 \text{ times}} = \frac{n}{2} \cdot \frac{n}{2} = \frac{n^2}{4}$

Let $k=1$, & $c = \frac{1}{4}$

$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n$

$\leq \underbrace{n + n + \cdots + n}_{n \text{ times}}$

$= n^2$

so let $c, k = 1$

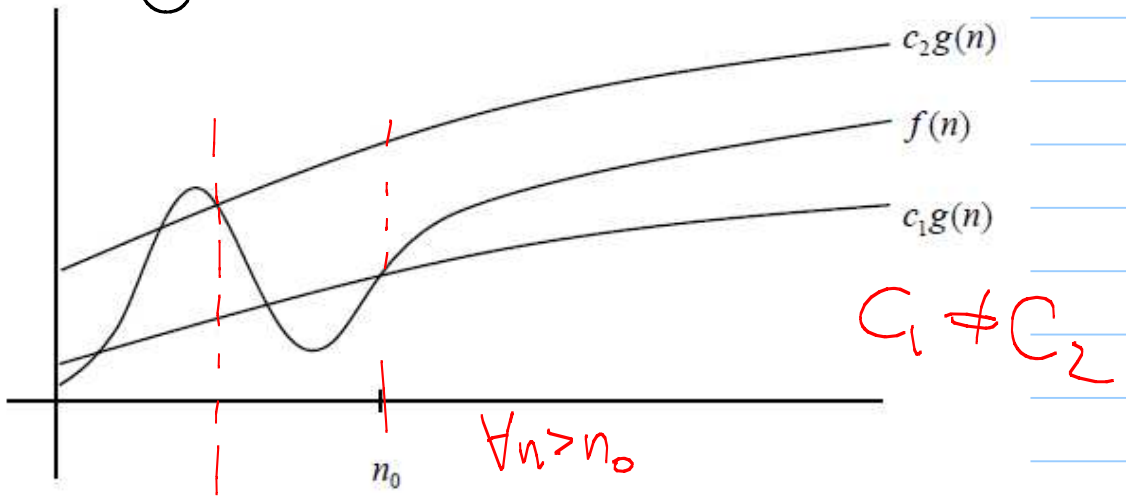
∎

Big-Theta

Let f & g be functions ($\mathbb{R} \to \mathbb{R}$ or $\mathbb{Z} \to \mathbb{R}$).
We say $f(x)$ is $\Theta(g(x))$ if

- $f(x)$ is $\Omega(g(x))$
- $f(x)$ is $O(g(x))$

We say f & g are asymptotically equivalent.



$C_1 \neq C_2$

Ex: $\sum_{i=1}^{n} i = \Theta(n^2)$.

Why?

We showed $\Omega(n^2)$ and $O(n^2)$