Math 135 : Induction

Note Title 1/22/2010

Announcements

- HW 1 due
- HW 2 out Monday, due 1 week later
- Worksheet today

Induction

A proof technique that is used to prove propositions of the form:

$$\forall n \geq 1, P(n)$$

Idea:

1. Show $P(1)$ true
2. Show $\forall k > 1$, $P(k-1) \rightarrow P(k)$

Since $P(1)$ is true (by 1):

$P(1) \rightarrow P(2)$ (by 2)

$P(2) \rightarrow P(3)$ (by 2)

$P(3) \rightarrow P(4)$ (by 3)

$\vdots$

$= 1+2+3+4+\dots+(n-1)+n$

Example: $\forall n \geq 1, \ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$P(n): \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

proof: by induction on n

① Show $P(1)$ is true

$\frac{n(n+1)}{2} = \frac{1(1+1)}{2} = 1$ $\quad \sum_{i=1}^{n} i = \sum_{i=1}^{1} i = 1$

So $P(1)$ is true, since $1=1$

② $\forall k > 1$, $P(k-1) \to P(k)$: $P(k-1)$ is: $\sum_{i=1}^{k-1} i = \frac{(k-1)(k-1+1)}{2} = \frac{(k-1)(k)}{2}$

$(k-1)k+2k = k^2-k+2k = k^2+k = k(k+1)$

So $\sum_{i=1}^{k} i = \left(\sum_{i=1}^{k-1} i\right) + k = \frac{(k-1)(k)}{2} + k = \frac{(k-1)k+2k}{2} = \frac{k(k+1)}{2}$

How to write inductive proofs

3 required parts

Base Case $P(1)$

Inductive Hypothesis

Assume $P(k-1)$ is true

Inductive Step

Use $P(k-1)$ to show $P(k)$

<u>Example</u>: Show that the sum of the first n odd integers is n^2.

$$\underbrace{1+3+\cdots+(2n-1)}_{n\ \#s} = \sum_{i=1}^{n} (2i-1) = n^2$$

<u>proof by induction</u>: <u>Base case</u>: $\sum_{i=1}^{1} (2i-1) = (2\cdot 1-1) = 1$

$n^2 = 1^2 = 1$ so they are equal if $n=1$

<u>IH</u>: Assume $\sum_{i=1}^{k-1} (2i-1) = (k-1)^2$

$$1 + 3 + 5 + \cdots + (2(k-1)-1) + (2k-1)$$

$$\parallel$$

<u>IS</u>: $$\sum_{i=1}^{k} (2i-1) = \underbrace{\sum_{i=1}^{k-1} (2i-1)}_{\text{apply IH}} + (2k-1)$$

$$= (k-1)^2 + (2k-1) = k^2 - 2k + 1 + 2k - 1$$

$$= k^2$$

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