Math 135 - Even More Induction

Note Title 1/27/2010

Announcements

- HW due next Monday
  (can submit in pairs)

- HW will be graded on Friday

Geometric Series:

$$\sum_{i=0}^{n} a \cdot r^i = \frac{ar^{n+1} - a}{r-1} \quad \text{when } r \neq 1$$

pf: by induction on n

Base case: $n=0$ $\quad \sum_{i=0}^{0} a \cdot r^i = a \cdot r^0 = a$ ✓

$$\frac{ar^{0+1} - a}{r-1} = \frac{ar - a}{r-1} = \frac{a(r-1)}{r-1} = a$$

IH: Assume $\displaystyle\sum_{i=0}^{n-1} a\cdot r^i = \frac{a\cdot r^n - a}{r-1}$

TS: $\displaystyle\sum_{i=0}^{n} a\cdot r^i = \underbrace{\sum_{i=0}^{n-1} a\cdot r^i}_{\text{Apply IH}} + a\cdot r^n$

$$= \frac{a\cdot r^n - a}{r-1} + a\cdot r^n = \frac{a\cdot r^n - a + a\cdot r^n(r-1)}{r-1}$$

$$= \frac{\cancel{a\cdot r^n} - a + a\cdot r^{n+1} - \cancel{a\cdot r^n}}{r-1} = \frac{a\cdot r^{n+1} - a}{r-1}$$

Strong induction:

We've been showing: $\forall n\, P(n)$

1. $P(1)$
2. $P(k-1) \rightarrow P(k)$

Strong induction is close, but here:

1. $P(1)$
2. $[P(1) \wedge P(2) \wedge \dots \wedge P(k-1)] \rightarrow P(k)$

We use more in our inductive step...

Ex: Show that if $n > 1$, then n can be written as a product of primes.

proof: Base case: $n = 2$ (which is prime!)

$2 = 1 \cdot 2$ ✓

IH: Any number k between 2 and $n-1$ can be written as the product of primes.

IS: Show that n can be written as product of primes.

Case 1: If n is prime, then done
$n = 1 \cdot n$ ✓

Case 2: If n is composite, then $n = a \cdot b$, where $2 \le a, b \le n-1$. By IH, a + b can be written as product of primes, so n can also. ∎

Ex: Prove that every amount of postage of 12 cents or more can be formed using just 4 & 5 cent stamps.