Math 135 - Binomial Theorem (5.4)

Note Title 4/12/2010

Announcements

- HW due Friday
- Everything is graded!
  (See me if missing anything, & feel free to email or come by to get your average so far.)

Last time:

Permutation $P(n,r)$: number of ways to order r things from a set of n

$C(n,r)$

Combinations $\binom{n}{r}$: number of ways to choose r things from a set of n

What is $(x+y)^3$?

$$(x+y)^3 = (x+y)(x+y)^2 = (x+y)(x^2+2xy+y^2)$$

Binomial Theorem

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

$$= \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \dots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$$

pf: combinatorial proof

LHS: polynomial: $\underbrace{(x+y)(x+y)\dots(x+y)}_{n \text{ times}}$

RHS: group terms in polynomial by powers of the y's.

$x^{n-j}y^j$ → we are choosing j of the y's. How many different ways to choose j of the y's? $\binom{n}{j}$

Ex: What is the coefficient of $x^{12}y^{13}$ in $(2x-3y)^{25}$?

$(2x-3y)^{25}$

$$\binom{25}{13}(2x)^{12}(-3y)^{13}$$

$$= \underbrace{\binom{25}{13}(2^{12})(-3)^{13}}\, x^{12}y^{13}$$

Another proof of: $\sum_{k=0}^{n} \binom{n}{k} = 2^n$

use binomial thm: $(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$

let $x = y = 1$

$$(1+1)^n = \sum_{j=0}^{n} \binom{n}{j} 1^{n-j} \cdot 1^j$$

$$2^n = \sum_{j=0}^{n} \binom{n}{j}$$

Other identities:

$$\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$$

binomial thm: $(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} \cancel{y^j}$

let $x = 1$, $\cancel{y} = 2$

$$(1+2)^n = \sum_{j=0}^{n} \binom{n}{j} 1^{n-j} 2^j$$

Rest of 5.4 is examples
of combinatorial proofs / identities.

On HW, I don't want to see
algebra!