Math 135: Discrete Math, Spring 2010

Sample First Midterm

Name:	Email:

1. This is a closed-everything exam. No notes or electronics of any kind are allowed.
2. Print your full name and your email address in the boxes above.
3. Print your name at the top of every page.
4. Please write clearly and legibly. If we can't read your answer, we can't give you credit.

Multiple Choice (5 points each)

Indicate your answers by circling the correct one. **Each question has exactly one correct answer.** There are no trick questions here, but you should still read everything carefully.

1. Which of the following subsets is in $\mathcal{P}(\mathcal{P}(\varnothing)) \times \mathcal{P}(\{1, 2, 3\})$? (Recall that we use $\mathcal{P}(A)$ to denote the powerset of a set A, and $\varnothing$ to denote the empty set.)

   (a) $\varnothing$

   (b) $(\varnothing, 1)$

   (c) $(\varnothing, \{1, 2\})$

   (d) $(\{\varnothing\}, 2)$

   (e) None of the above

2. Which of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$ is not a bijection?

   (a) $f(n) = n - 1$

   (b) $f(n) = n^2 + 1$

   (c) $f(n) = n^3$

   (d) $f(n) = 3n + 11$

   (e) None of the above

3. Which of the following is the converse of the statement, "You are crazy if you are taking Math 135."

   (a) If you are crazy, then you are taking Math 135.

   (b) If you are not taking Math 135, then you are not crazy.

   (c) If you are not crazy, then are you not taking Math 135.

   (d) You are taking Math 135 and you are not crazy.

   (e) Butterflies are pretty.

4. Let A be the set of computer science majors and B be the set of classes. Consider the following predicates:

   - $P(x)$ means "x is stinky"
   - $Q(x)$ means "x has showered"
   - $R(x, y)$ means "x sits next to me in class y"

   Which of the following is equivalent to the statements "In every class I take, there is a computer science student sitting next to me in that class who has not showered and is stinky."

   (a) $\exists x \in B, \forall y \in A, R(x, y) \vee P(x)$
   (b) $\forall x \in B, \exists y \in A$, if $R(x, y)$ then $\not Q(x)$
   (c) $\exists x \in B, \forall y \in A, R(x, y) \vee P(x) \vee \not Q(x)$
   (d) $\forall x \in B, \exists y \in A, R(y, x) \wedge P(x) \wedge \not Q(x)$
   (e) None of the above.

5. Which of the following is not a *member* of the set $\mathcal{P}(\{4, 5\}) \times \{1, 2, 3\} \times \{\pi, e\}$?

   (a) $(4, \{1, \pi\})$
   (b) $(\emptyset, 2, \pi)$
   (c) $(\{4, 5\}, 2, e)$
   (d) $(\{4\}, 3, \pi)$
   (e) None of the above.

6. Let $A(x)$ be the predicate "x is honest", and let $B(x)$ be the predicate "x is a politician". Let S be the set of people in the world. Which of the following is equivalent to the *negation* of the statement: $\forall x \in S$, if $B(x)$ then $\neg A(x)$.

   (a) Every person who is a politician is honest.
   (b) There is a person who is a politician and is honest.
   (c) All honest politicians are people.
   (d) There is a person who, if he or she is not honest, then they are a politician.
   (e) There is a person who is a politician or is not honest.

Short Answer (10 points each)

Write your solutions in the space provided.

7. Let $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and $C = \mathcal{P}(\varnothing)$.

   (a) What is $(B - A) \cup (C \cap A)$?

   (b) What is $(A \cap B) \cup \mathcal{P}(A)$?

8. Prove that for any integer $n \geq 0$, if n^2 is even, then n is even.

Long Answer (15 points each)

Write your solutions in the space provided. (If you run out of room, you may continue your answer on another page, but please tell us where to look!)

9. Prove that for all $n \geq 0$, $\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$.

10. Suppose we have two functions $f : A \to B$ and $g : B \to C$, and are given that g and $g \circ f$ are onto. Prove that f is *not* necessarily onto.

11. Prove by induction that any positive integers can be written as a sum of *distinct* powers of 2. 'Distinct' means that each power of 2 appears at most once in the sum. For example:

$$4 = 2^2 \qquad 17 = 2^4 + 2^0 \qquad 23 = 2^4 + 2^2 + 2^1 + 2^0 \qquad 173 = 2^7 + 2^5 + 2^3 + 2^2 + 2^0$$

In other words, prove that any positive integer can be written in binary!

[Hint: In the inductive case of your proof, identify a power of 2 that must appear in the sum.]