Math 135: Discrete Mathematics, Spring 2010

Sample Final Exam

Name:	Email Address:

1. This is a closed-everything exam. No notes or electronics of any kind are allowed. However, you can find a 3-page 'cheat sheet' at the end of this exam booklet.
2. Print your full name and your email address in the boxes above.
3. Print your name at the top of every page.
4. Please write clearly and legibly. If we can't read your answer, we can't give you credit.
5. You have **two hours** to complete the exam. Remember that problems are not necessarily in order of difficulty, so pace yourself!

Questions (10 points each)

1. Recurrences:

   (a) Let $T(n) = 6T(\frac{n}{6}) + n^2$. Solve the recurrence $T(n)$ *asymptotically*. Show your work.

   (b) Give the *general form* of the solution to the following recurrence: $A(n) = A(n-1) + 2A(n-2) + n - 3$. Show your work.

2. Prove that $\sum_{i=1}^{n} i \cdot i! = (n+1)! - 1$ whenever n is a positive integer.

3. (a) How many different 3 letter initials are there that begin with an A?

   (b) How many ways can a photographer at a wedding arrange 6 people in a row if the bride must be next to the groom?

   (c) There are 10 questions on a discrete math final exam. How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question must be worth at least 5 points?

4. Show that an edge in a graph is a cut edge if and only if the edge is not part of any cycle in the graph.

5. Prove that if G is a graph with n vertices and fewer than n edges, then G contains a vertex of degree ≤ 1. (Hint: Think degree-sum formula.)

6. Determine whether the following functions from $\mathbb{R}$ to $\mathbb{R}$ are 1-1 or onto:

   (a) $f(x) = x^2 + 7$

   (b) $f(x) = -3x + 4$

7. Let S be a subset of $\{1, 2, \ldots, 3n\}$ which contains $2n + 1$ numbers. Show that S contains 3 consecutive integers.

   Hint: Use pigeonhole.

8. Analyze the asymptotic complexity of the following algorithm:

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Procedure MyALG(A[1..n]):
  x := 0
  for i := 1 to n
      for j := 1 to i
          x := x + A[i]
  return x
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9. Decide if the following properties hold for sets. You do not need to prove your answers. (Hint: Use Venn diagrams.)

   (a) $A \cap (B \cup C) = (A \cap B) \cup C$

   (b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   (c) $(B \cap C) - A = B \cap (C - A)$

10. Thirteen people on a softball team show up for a game.

(a) How many ways are there to choose 10 players to take the field?

(b) Of the 13 people who show up, 3 are women. How many ways are there to choose 10 players to take the field if at least one of them must be a woman?

(c) In this group of players, there are 2 sets of identical twins and one set of identical triplets. How many ways are there to line up all 13 players for a picture after the game if you cannot tell the identical siblings apart?

11. Find asymptotic bounds for the following functions which are as tight as possible:

(a) $f(x) = 2x^3 + x^2 \log_2 x^2$

(b) $f(x) = (x^4 + 3x)(\log_3 x + x)$

(c) $f(x) = (x/2 + 64 \cdot 2^x)(5x^2 - 5x)$

(scratch paper)

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