Math 135: Discrete Mathematics, Spring 2010
Homework 8

Due *in class* on April 23, 2010

For this homework, you may write up solutions with 1 partner; both of you will receive the same grade based on your joint writeup.

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1. Answer the following questions - and be sure to explain your answers.

   (a) How many different ways are there to choose a dozen donuts from the 21 varieties at a donut shop?

   (b) How many different combinations of pennies, nickels, dimes, quarters, and half dollars can a piggie bank have which contains 20 coins?

   (c) How many solutions are there to the inequality $x_1 + x_2 + x_3 \leq 11$, where each x_i is a nonnegative integer?

   (d) How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 98$, where each x_i is an integer ≥ 1?

   (e) How many different strings can be made from the letters in the word MISSOURI?

   (f) A professor packs her collection of 40 different issues of a mathematics journal into 4 boxes with 10 issues per box. How many ways are there to distribute the journals if each box is numbered (so they are distinguishable)?

   (g) The same professor now packs her collection of 40 different issues of a mathematics journal into 4 boxes with 10 issues per box. How many ways are there to distribute the journals if the boxes are identical (so they are indistinguishable)?

2. (a) Pascal's identity states that:

   $$\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$$

   Prove this identity using the algebraic formula for $\binom{n}{r}$.

   (b) Prove the following identity holds for any $n \geq k$ via an induction proof:

   $$\sum_{i=k}^{n} \binom{i}{k} = \binom{n+1}{k+1}$$

   [Hint: In your inductive step, you'll need to use Pascal's identity from part a.]