

Math 135: Discrete Mathematics, Spring 2010

Homework 7

Due *in class* on April 16, 2010

For this homework, you may write up solutions with 1 partner; both of you will receive the same grade based on your joint writeup.

1. Answer the following questions - and be sure to explain your answers.

- (a) The name of a variable in the C programming language is a string that can contain uppercase letters, lowercase letters, digits, or underscores. Further, the first character must be a letter (either upper or lower case) or an underscore. If the name of a variable is determined only by its first 8 characters (the rest are ignored), how many different variables can be named in C?
- (b) How many ways are there for 10 women and 6 men to stand in a line so that no two men stand next to each other?
- (c) A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them. How many balls must she select in order to be sure of having at least 3 balls of the same color?
- (d) A professor write 40 discrete math True/False questions for a text. If exactly 17 of these problems are true and questions can go in any order, how many different answer keys are possible?
- (e) Seven women and 9 men are on the faculty of a math department. How many ways are there to select a committee of 5 members of the department if at least one woman must be on the committee? What if at least one woman and one man must be on the committee?
- (f) How many bit strings of length 10 contain either 5 consecutive 1's or 5 consecutive 0's?
- (g) What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$?

2. Give combinatorial proofs of the following identities:

- (a) $\binom{n}{r} \binom{r}{k} = \binom{n}{k} \binom{n-k}{r-k}$
- (b) $\sum_{k=0}^n 2^k = 2^{n+1} - 1$
- (c) $\binom{2n}{2} = 2 \binom{n}{2} + n^2$

3. Show that if $n + 1$ integers are chosen from the set $\{1, 2, \dots, m \cdot n\}$, then there are always two which differ by at most $m - 1$.