Math 135: Discrete Mathematics, Spring 2010
Homework 4

Due *in class* on Monday, March 1, 2010

For this homework, you may write up solutions with 1 partner; both of you will receive the same grade based on your joint writeup. Please make sure to read the course policies on homework *before* writing up your homework.

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1. Show that x^3 is $O(x^4)$ but that x^4 is not $O(x^3)$.

2. Show that if a and b are real numbers with $a > 1$ and $b > 1$, if $f(x)$ is $O(\log_b(x))$, then $f(x)$ is $O(\log_a(x))$.

3. Give a big-O estimate (as tight as possible) of the following function. Be sure to justify your answer, either using theorems from class or a direct big-O proof.

$$f(x) = (\pi + (e^{10!}))^7 + \sum_{n=0}^{10} \left(\frac{1}{e} + n\right)^{25} x^n$$

4. Sort the following functions from asymptotically smallest to asymptotically largest. Include a proof for each relationship. To simplify your answers, write $f(n) \ll g(n)$ to mean $f(n) = O(g(n))$, and write $f(n) \equiv g(n)$ to mean $f(n) = \Theta(g(n))$.

   For example, the functions n^3, $2n$, and $2^{\log_2 n}$ could be sorted as $2n \equiv 2^{\lg n} \ll n^3$ or as $2^{\lg n} \equiv 2n \ll n^3$. Further, you would need to include three proofs: that $2n = O(2^{\lg n})$, that $2n = \Omega(2^{\lg n})$, and that $2n = O(n^3)$.

$$\frac{n}{\lg n} \qquad 8n^3 + 10n + 1,024 \qquad n\lg(n^4) \qquad 2^{3\log_2 n} \qquad \frac{n}{n!}$$

5. Suppose that $f(x)$ is $O(g(x))$. Does it follow that $2^{f(x)} = O(2^{g(x)})$? Prove your answer.