Math 135: Discrete Mathematics, Spring 2010
Homework 2

Due *in class* on Friday, Feb. 12, 2010

For this homework, you may write up solutions with 1 partner; both of you will receive the same grade based on your joint writeup. Please make sure to read the course policies on homework *before* writing up your homework.

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1. Prove or disprove the following:
   (a) $A \times (B \cup C) = (A \times B) \cap (A \times C)$
   (b) $(A - C) \cap (C - B) = \emptyset$
   (c) If $\mathcal{P}(A) = \mathcal{P}(B)$, then $A = B$.
   (d) If $A \cap C = B \cap C$, then $A = B$.

2. (a) Prove that if A and B are sets, then $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.
   (b) Give a counterexample for the following statement: if A and B are sets, then $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$.

3. If A and B are sets, then the *symmetric difference* of A and B is written $A \triangle B$ and is defined to be $(A - B) \cup (B - A)$. Prove that if $A_1, A_2, \ldots, A_n$ are sets, then

$$(((A_1 \triangle A_2) \triangle A_3) \cdots \triangle A_{n-1}) \triangle A_n = \{x \ : \ x \text{ is in an odd number of the sets } A_1, A_2, \ldots, A_n\}.$$

   Hint: Think induction!

4. For the following functions, decide whether each one is one-to-one, onto, and bijective, and prove each of your answers.
   (a) $f : \mathbb{R} \to \mathbb{R}$, with $f(x) = x^2 + 1$
   (b) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, with $f(m, n) = 2m - n$
   (c) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, with $f(m, n) = m^2 - n^2$

5. Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ which is:
   (a) one-to-one but not onto
   (b) onto but not one-to-one
   (c) both onto and one-to-one (but NOT the identity function)
   (d) neither one-to-one or onto

6. Let f be a function from the set A to the set B. Let S and T be subsets of A.

   (a) Prove that $f(S \cup T) = f(S) \cup f(T)$

   (b) Prove that $f(S \cap T) \subseteq f(S) \cap f(T)$

   (c) Give an example to show that the inclusion from part (b) may be proper - in other words, give examples of sets and a function where $f(S \cap T) \subset f(S) \cap f(T)$