Math 135: Discrete Mathematics, Spring 2010
Homework 1

Due *in class* on Friday, January 22, 2010

Submit your solutions for this homework *in class* on Friday, January 22. For this homework, you may write up solutions with 1 partner; both of you will receive the same grade based on your joint writeup. Please make sure to read the course policies on homework *before* writing up your homework.

---

1. Write the negation, contrapositive, converse and inverse of the following statements.

   (a) For any integer x, if x is even, then x^2 is even.

   (b) You will be late to class if you oversleep.

   (c) If Dr. Chambers' daughter is sick this week, then she either brings her to lecture or gets a babysitter.

2. Rewrite the following propositions as unambiguous English sentences, given the following prepositions.

   - $A(x)$ means "x likes West Wing".
   - $B(x)$ means "x likes Buffy".
   - $C(x)$ means "x has good taste".
   - $D(x)$ means "x watches TV".

   For example the statements $\forall x[D(x) \rightarrow A(x)]$ could be translated to "Everyone who watches TV likes West Wing."

   (a) $\forall x[A(x) \vee B(x) \rightarrow D(x)]$

   (b) $\exists x[A(x) \wedge B(x)]$

   (c) $\forall x[B(x) \rightarrow C(x)]$

   (d) $\exists x[A(x) \vee B(x) \wedge C(x) \wedge D(x)]$

3. Express the **negations** of the following statements so that all negation symbols appear immediately preceding the predicates (and not outside any quantifiers or groups of predicates).

   (a) $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$

   (b) $\exists x \exists y (Q(x, y) \rightarrow Q(y, x))$

   (c) $\forall y \exists x \exists z (T(x, y, z) \vee Q(x, y)) \wedge \exists x \forall y \neg P(x, y)$

4. Classify the following formulas into logically equivalent groups. (Hint: Try using truth tables!)

   (a) p

   (b) $p \vee \neg p$

   (c) $p \wedge \neg p$

   (d) $(p \wedge q) \rightarrow p$

   (e) $(p \vee q) \rightarrow p$

   (f) $((p \vee q) \wedge \neg q) \rightarrow (p \wedge q)$

   (g) $(p \vee \neg p) \rightarrow (p \wedge \neg p)$

   (h) $(((p \vee q) \vee \neg q) \vee (r \wedge p)) \wedge (p \vee \neg q)$

5. While walking across campus, you come across 3 people have an argument. The first, Alice, tells you, "Bob or Carol is lying." The second, Bob, tells you, "Carol is lying". The third, Carol, tells you, "Alice and I are both telling the truth". Who, if anyone, is telling you the truth?

6. **Extra Credit:** Next, you come across two different people on your walk. Donald says, "I am lying if Erik is." Erik says, "If I am lying, then Donald is lying." Can you tell who if anyone is telling the truth?