In class exercises: Logic, proofs, and sets

1. **Implications.** Consider the statement:

   If an integer n is divisible by 6, then n is divisible by 3.

   (a) Write its converse:

   (b) Write its contrapositive:

   (c) Which of the three statements are true? Prove your answers.

2. **Quantifiers.** For each of the following statements, write its negation. Do not use the phrase "it is not the case that." Then determine whether the original statement is true or false.

   (a) Every integer is either positive or negative.

   (b) There exists an integer n such that $n^2 = 2$.

   (c) For every integer n, there exists an integer m such that $m > n$.

3. **Sets.** Suppose

   $$A = \{1, 2, 4\}, \qquad B = \{2, 3, 4\}, \qquad C = \{1, 3, 5\}.$$

   Compute

   $$(A \cup B) \cap C \qquad \text{and} \qquad (A \cap C) \cup (B \cap C).$$

   What do you notice?

4. **Proof by contrapositive.** Prove that if n^2 is even, then n is even.

5. **Proof by contradiction.** Prove that there do not exist integers a and b such that

$$2a + 4b = 7.$$

6. **Proof about sets.** Prove that for any sets A and B,

$$A \cap B \subseteq A.$$