

In class exercises: More induction!

1. A *rooted binary tree* is defined recursively as follows:

- A single vertex is a rooted binary tree.
- If T is a rooted binary tree, then a new root may be attached above the root of T .
- If T_1 and T_2 are rooted binary trees, then a new root may be attached above the roots of T_1 and T_2 .

Prove by structural induction that the vertices of every rooted binary tree can be colored black and white so that adjacent vertices receive different colors.

2. Define a collection $\mathcal{S}$ of binary strings recursively as follows:

- The empty string ϵ belongs to $\mathcal{S}$.
- If $w \in \mathcal{S}$, then $0w1 \in \mathcal{S}$ and $1w0 \in \mathcal{S}$.
- If $u, v \in \mathcal{S}$, then the concatenation uv belongs to $\mathcal{S}$.

Prove by structural induction that every string in $\mathcal{S}$ contains the same number of 0's as 1's.