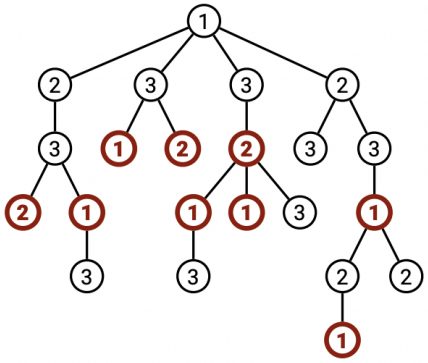
DP on trees

1. Since so few people came to last year's holiday party, the president of Giggle decides to give each employee a present instead this year. Specifically, each employee must receive on the three gifts: (1) an all-expenses-paid sixweek vacation anywhere in the world, (2) an all-the-pancakes-you-can-sort breakfast for two at Jumping Jack Flash's Flapjack Stack Shack, or (3) a burning paper bag full of dog poop. However, corporate regulations prohibit any employee from receiving exactly the same gift as his/her direct supervisor. Any employee who receives a better gift than his/her direct supervisor will almost certainly be fired in a fit of jealousy. As Giggle's social party czar, it's your job to decide which gift each employee receives.

   More formally, you are given a rooted tree T, representing the company hierarchy, stored as a data structure where you have saved the root as $T.root$. For each node v in the tree, there is a list of children saved in an array. You want to label the nodes of T with integers 1, 2, or 3, so that every node has a different label from its parent. The cost of an labeling is the number of nodes with smaller labels than their parents.

   See below for an example of such a tree, which has cost 9, where all fired employees are shown in bold (since they have values lower than their direct supervisor.)



   (a) Give a recursive formulation to calculate the minimum cost labeling rooted at a node v.

   (b) Design and analyze an algorithm to compute the minimum-cost labeling of T, so that the fewest number of people are fired.