Transition: DS + Algs

Intro
Logic

First: Intro + overview

- Syllabus (updating)
  ↳ HW vs Quizzes?
- Survey (by Wednesday if possible)
- Class structure: readings + lecture/discussion
- In 356 C Fitz starting Wed.

Perusall → \$5 : access via Canvas
↳ 1 for Wed, 2 on Friday

Perusall:

60% : reading completion

60% : reading "engagement"

20% : comments

20% : questions / reactions

20% : opening

→ = 240%

ask if confusing!

Course Goals

- Needed discrete math:
  - ↳ programming / algs
  - proofs & reasoning, big-O
- Data structures:
  - analysis
- Algorithms:
  - 1/2 of ugrad topics
- Complexity

Dfn: A proposition is a declarative statement which is either true or false.

Ex:
- This is transition to computing.
- I am a professor.

Non-examples:

- Spiders are creepy.
- Go pick that up.

Note: English can be tricky!

Operations on propositions

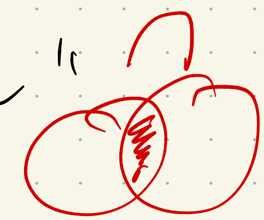
Let p be a proposition.

ie: p = "The sky is blue"

- **Negation** of p: $\neg p$ — 1 or 0

"It is not the case that p"

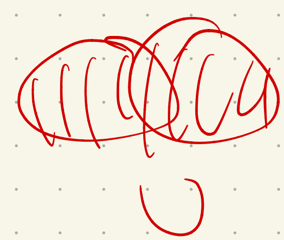
or, shorter: "The sky is not blue"



- **Conjunction**: $p \wedge q \rightarrow p$ and q

True if both p and q are true.

- **Disjunction**: $p \vee q \rightarrow p$ or q



True if either p or q is true.

Truth tables

Summary of truth values of compound propositions for any combination of inputs.

Example:

p	q	$p \vee q$	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	T	F	F	F
T	F	T	F	F	T	T
F	T	T	F	T	F	T
F	F	F	F	T	T	T

Why? Can help break down complex situations, & can help with logical equivalence ie

$\neg p \vee \neg q$ vs $p \wedge q$

Other concepts

- Tautology: a proposition that is always true
- Contradiction: a proposition that is alway false

Examples:

$p \lor \neg p$: $= T$

$p \land \neg p$: $= F$

Table:

p	$\neg p$	$p \lor \neg p$	$p \land \neg p$
T	F	T	F
F	T	T	F

- **Implications**: $\boxed{p \to q}$ i.e.
  - "If p, then q"
  - "p implies q"
  - "q if p"

Ex: "If I am elected, then I will lower taxes."

I am not elected or I will lower taxes.

When is this false?

p	q	$\boxed{p \to q}$	$\neg p \vee q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

p true but q not

"vacuously true"

Consider $p \to q$:
- Converse: $q \to p$
- Inverse: $\neg p \to \neg q$
- Contrapositive: $\neg q \to \neg p$.

<u>Example</u>: p = "The number 15 is prime"
q = "15 has no divisors"

- $\overset{=T}{p \to q}$: If 15 is prime, then it has no divisors.
- $\overset{=T}{q \to p}$: "If $\underbrace{\text{15 has no divisors}}_{F}$, then $\underbrace{\text{it is prime}}_{F}$."
- $\overset{=T}{\neg p \to \neg q}$: "If $\underbrace{\text{15 is not prime}}_{F}$, then it has divisors."
- $\underset{=T}{\neg q \to \neg p}$: "If 15 has divisors, then 15 is not prime",

Truth table

p	q	$p \to q$	$q \to p$	$\neg p$	$\neg q$	$\neg p \to \neg q$	$\neg q \to \neg p$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

vacuous

"vacuous!!"

Which are equivalent?

$$p \to q \equiv \neg q \to \neg p$$

$$q \to p \equiv \neg p \to \neg q$$

Using logical equivalence

We can develop some useful "laws" using logical equivalence:

Example: De Morgan's Law

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T					
T	F					
F	T					
F	F					

Others

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee \mathbf{F} \equiv p$	$p \wedge \mathbf{T} \equiv p$
Domination laws:	$p \wedge \mathbf{F} \equiv \mathbf{F}$	$p \vee \mathbf{T} \equiv \mathbf{T}$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \wedge \neg p \equiv \mathbf{F}$ $\neg \mathbf{T} \equiv \mathbf{F}$	$p \vee \neg p \equiv \mathbf{T}$ $\neg \mathbf{F} \equiv \mathbf{T}$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities:	$p \rightarrow q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Predicates

propositions which depend on a variable
(or possibly several)

Written $P(x)$

ie $P(x) =$ "x is > 0"

$Q(x,y) =$ "x is a teacher of class y"

$R(x) =$ "x is prime"

Truth value depends on variables:

ie: $P(5) =$

$P(-2) =$

$Q($"Erin", "CSE 50502"$) =$

Quantifiers

- Universal quantifier:

  $\forall x\, P(x)$

  ↳ For all x in $\underbrace{\text{domain}}$, $P(x)$ is true

Example:

$\forall x \in \mathbb{Z}$, x is prime.

Counterexample:

To show false, find one element in domain where $P(x)$ is false

- Existential quantifier

  $\exists x\ P(x)$

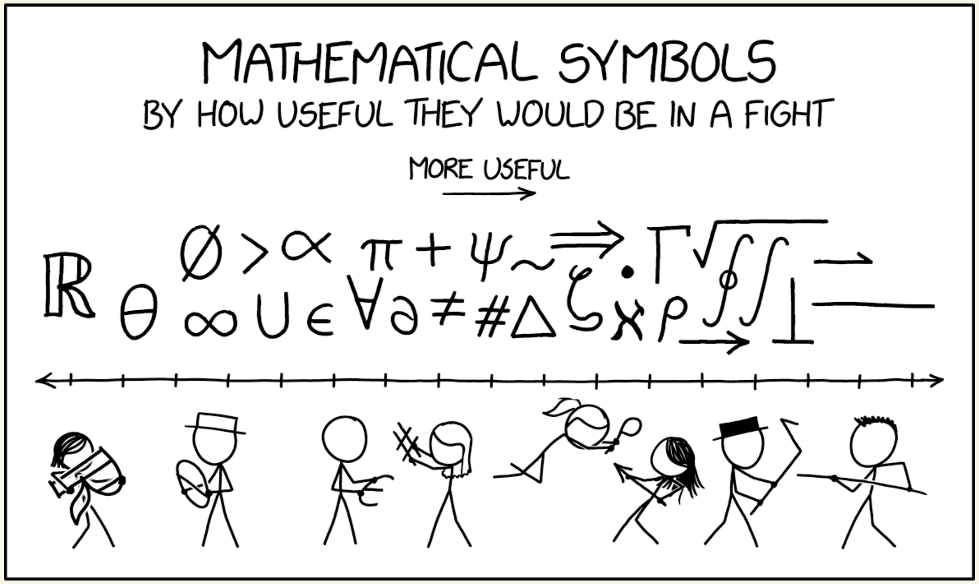
  ↳ "There exists a value x in the domain, such that $P(x)$ is true

Example! $\exists x \in \mathbb{Z}$ s.t. x is prime.

How to show it's true?

Quick note

At this point, let me pause and acknowledge that math notation can be a lot.



Please stop me with questions!

Using Logical operators

Can combine predicates using all the same logical operators:

$P(x) =$ " $x+1>2$ "

$Q(x) =$ " $x<2$ "

Example:

$\forall x \in \mathbb{R},\ P(x) \vee \neg Q(x)$.

Translate:

Ordering + combining

In short: order matters!

Let $Q(x,y) =$ "$x+y=0$"

Translate

- $\forall x \exists y$ s.t. $Q(x,y)$:

- $\exists x \forall y$ s.t. $Q(x,y)$:

Negations

Consider $P(x) =$ "x has taken programming"

$\forall$ people x, $P(x)$:

What is $\neg(\forall x P(x))$?

Reverse it: what is $\neg(\exists x P(x))$?

They "stack", plus can use laws:

- $\neg(\forall x \forall y\ P(x,y))$:

- $\neg(\forall x\ (P(x) \vee Q(x)))$:

Implications + quantifiers

Let $P(x) = $ "$x > 0$"

and $Q(x, y) = $ "$xy \leq 0$"

translate:

$$\forall x \forall y \ (P(x) \wedge \neg P(y)) \rightarrow Q(x,y)$$

Proofs

A **theorem** (or **lemma**, **proposition**, etc.) is a statement that can rigorously be shown to hold true.

Generally in math, something like:

"If n is even, then it is divisible by 2."

"$\forall \varepsilon > 0 \; \exists \delta > 0$ s.t. if $|f(x) - f(y)| < \varepsilon$, then $|x - y| < \delta$".

The sequence of statements giving the argument is a **proof**.