CSE/AMCS 60973: Topological Data Analysis Homework 1

This assignment is due on Sept 8, by 11:59pm, and covers the material in chapter 1 of our textbook by Dev and Wang.

Note that on this (or any) homework, you are welcome to study and work in groups. In fact, I highly encourage you to do so! Please also stop by my office to say hello and tell me how it's going, or let me know if you have questions or could use help on any background. Likewise, you're welcome to use google or any other resource. I do request the following:

- Include an acknowledgement section, listing any students you worked with or other resources you used including chatgpt!
- Type up your own solution. I encourage you to use latex, since it's a necessary tool in our field.
- Make sure you type up your own solutions, as verbatim copying will be given a 0.

Required Problems

Do 3 out of the following 7 problems. For any definitions or ideas we didn't cover in class, be sure to explain and define. Problems are taken from Dey Wang (DW), unless they are explicitly written out.

- 1. DW 1.6.1. Be sure to explain why the constructions you have created are Hausdorff.
- 2. DW 1.6.2
- 3. DW 1.6.7
- 4. DW 1.6.10
- 5. DW 1.6.12
- 6. Recall the Jordan Curve Theorem says that every simple closed curve in the plane decomposes \mathbb{R}^2 into two pieces.
 - (a) Show the same is true for any simple closed curve on the 2-sphere:

$$S^2 = \{ x \in \mathbb{R}^3 \text{ such that } ||x|| = 1 \}$$

- (b) Give an example that shows the result does not hold for simple closed curves on the torus.
- 7. Is every graph that can be embedded on the Möbius strip planar? Either prove your answer or give a counterexample.