Algorithms

Recursion

Today

- Syllabus!
- Comments on Perusall?
- Change in office hours today. (apologies!)

11-12 today

1-2 tomorrow

or email / stop in

Reading:

- Towers of Hanoi:

$$H(n) = 2H(n-1) + 1$$

or $a_n = 2a_{n-1} + 1$

- Mergesort:

$$M(n) = 2M\left(\frac{n}{2}\right) + O(n)$$

$$m_n = 2m_{n/2} + n$$

Master-thm

- Quicksort: only good if "random" pivot

$$Q(n) \leq 1 \cdot Q(n-1) + n$$

quadratic $O(n^2)$

unroll ↳

$$Q(n) \leq n + Q(n-1) = n + (n-1) + Q(n-2)$$

$$= n + (n-1) + (n-2) + Q(n-3) = \cdots$$

$$O(n^2) = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \binom{n}{2} =$$

Recursion trees: Master theorem

One way to tackle recurrences.

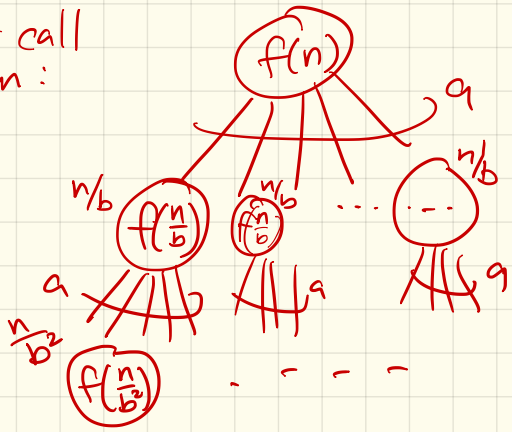
(There are others!)

Big idea: $T(k) = aT\left(\frac{k}{b}\right) + f(k)$

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

Sum total amount of "work":

Root - first call of fcn:



level k: $T(1)$

Example:

$T(n) =$