Algorithms

NP-Hardness: Some final reductions

Recap

- HW8 due
- HW9 next Wed.
- Final worksheet, w/ 1 problem on final
- Reading next week: every day.

Last time:

Graph reductions:

- Ind Set
- Clique
- Vertex Cover

In book:

- Hamiltonian cycle
- Traveling salesman

Subset Sum:

Given a set of numbers
$X = \{x_1, x_2, x_3, \dots, x_n\}$
and a target t, does some subset of X sum to t?

Ex: (actually did this one! See lecture from Ch. 2

Runtime:

Well, x_i is either in subset or not.

$\Rightarrow$ Dyn. programming

exponential

Subset Sum is NP-Hard.

Reduction: Vertex Cover

Input: Graph G & size k.

Q: Is there a set of vertices of size k that "cover" all edges.

Construct a set of numbers:

Reminder: Base 4 #s

_ _ _ _ _

0-3

$\underline{1}\,\underline{2}\,\underline{3} = 3 + 2\cdot 4^1 + 1\cdot 4^2$

Take each edge of G & give it a # in base 4:

Put numbers into set

edge 1: 1

edge 2: 10

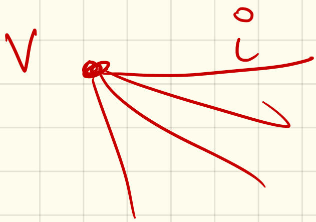
edge 3: 100

⋮

edge E: $\underbrace{10\cdots0}_{E}$

Currently, have E #s.

Now, for each vertex, add a # to set.



Put 1's if e is adjacent to

$$1 \quad \underbrace{\qquad 1 \qquad}_{E}$$

i

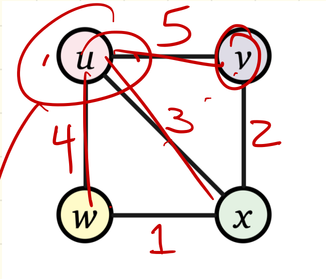
Add V more #s.

Target: $t = 4^E \cdot k + \sum_{i=0}^{E-1} 4^i \cdot 2$

reduction cont:

Ex:

$k=2$



$$\cancel{\begin{aligned} a_u &:= 111000_4 = 1344 \\ a_v &:= 110110_4 = 1300 \\ a_w &:= 101101_4 = 1105 \\ a_x &:= 100011_4 = 1029 \end{aligned}}$$

$$\begin{aligned} b_{uv} &:= 010000_4 = 256 \\ b_{uw} &:= 001000_4 = 64 \\ b_{\cancel{vw}\,ux} &:= 000100_4 = 16 \\ b_{vx} &:= 000010_4 = 4 \\ b_{wx} &:= 000001_4 = 1 \end{aligned}$$

1 number per edge

$a_u = 111100$

$a_v = 110010$

$$t = \underbrace{4^5 \cdot \overset{k}{2}} + \sum_{i=0}^{4} 4^i \cdot 2$$

Thm: Vertex Cover $\Leftrightarrow$ Subset Sum

Proof:

$\Rightarrow$ suppose have VC of size k:
choose those a_i's to be in subset.

Sum: most significant digit
$\Rightarrow k \cdot 4^E$

other digits: $= 2 \cdot 4^i$
(since each edge is covered)

$\Leftarrow$: Suppose have subset
$= k \cdot 4^E + \sum_{i=0}^{E-1} 2 \cdot 4^i$

$\hookrightarrow$ must have chosen exactly k vertex #'s

Another: Partition

Given a set of n numbers, can you partition into 2 sets X + Y so that

$$\sum_{x \in X} x = \sum_{y \in Y} y \;?$$

Easy reduction:

Reduce subset sum to 2-partition:

Given $X = \{x_1 \dots x_n\}$ + t.

Hint: Let $S = \sum_{i=1}^{n} x_i$

Create X': $X' := X \cup \{S+t, 2S-t\}$

$= \{x_1, \dots, x_n, S+t, 2S-t\}$

$$X' = \{\underbrace{x_1, \ldots, x_n}_{=S}, S+t, 2S-t\}$$

Claim: X has subset $= t$

$\iff$ X' can be partitioned

$\Leftarrow$ Suppose X' can be partitioned.

2 parts $\left[S+t\ \underbrace{\quad}_{=S-t}\right]$ $\left[2S-t\ \underbrace{\quad}_{=t}\right]$

$\Rightarrow$ some subset sums to t!

$\Rightarrow$ Sppse subset $= t$

Build 2 halves of X':

Set Cover:

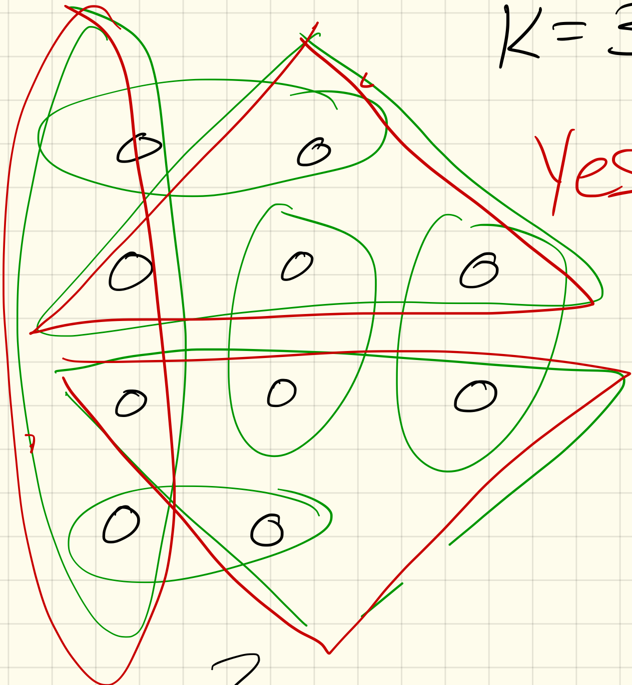
Given a set U of n elements, a collection $S_1, S_2, \ldots, S_m$ of subsets of U, + a number k, is there a collection of k of the S_i's whose union is all of U?

$K = 3$?

Ex:

elements in U:

Subsets $S_1, \ldots, S_7$



Yes

Answer?

Set Cover is NP-Hard:

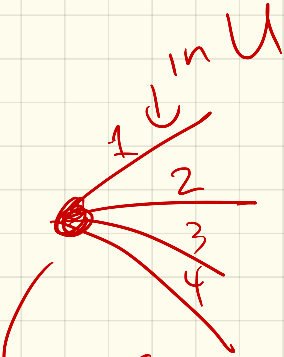
Reduction from vertex cover, so input is G & k.

Construct:

$U = \{ \text{edges } uv \in G \}$

S_i's: For each vertex, $S_i = \{ \text{edges } v_i \text{ connects to} \}$

& k: $k = k$



$\{1, 2, 3, 4\}$

Vertex cover of size k

$\iff$ Set cover of size k

Some fun examples

arXiv.org > cs > arXiv:1203.1895

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Computer Science > Computational Complexity

Classic Nintendo Games are (Computationally) Hard

Greg Aloupis, Erik D. Demaine, Alan Guo, Giovanni Viglietta

(Submitted on 8 Mar 2012 (v1), last revised 8 Feb 2015 (this version, v3))

We prove NP-hardness results for five of Nintendo's largest video game franchises: Mario, Donkey Kong, Legend of Zelda, Metroid, and Pokemon. Our results apply to generalized versions of Super Mario Bros. 1-3, The Lost Levels, and Super Mario World; Donkey Kong Country 1-3; all Legend of Zelda games; all Metroid games; and all Pokemon role-playing games. In addition, we prove PSPACE-completeness of the Donkey Kong Country games and several Legend of Zelda games.

Comments: 36 pages, 36 figures. Fixed some typos. Added NP-hardness results (with proofs and figures) for American SMB2 and Zelda 2

Subjects: **Computational Complexity (cs.CC)**; Computer Science and Game Theory (cs.GT)

Cite as: **arXiv:1203.1895 [cs.CC]**
(or **arXiv:1203.1895v3 [cs.CC]** for this version)

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From: Alan Guo [view email]
[v1] Thu, 8 Mar 2012 19:37:20 GMT (627kb,D)
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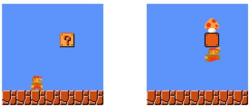
[v3] Sun, 8 Feb 2015 19:45:26 GMT (3425kb,D)

Which authors of this paper are endorsers? | Disable MathJax (What is MathJax?)

Link back to: arXiv, form interface, contact.



Left: Start gadget for Super Mario Bros. Right: The item block contains a

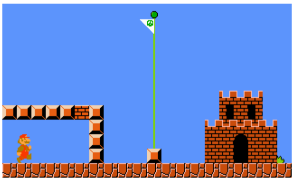


Figure 9: Finish gadget for Super Mario Bros.

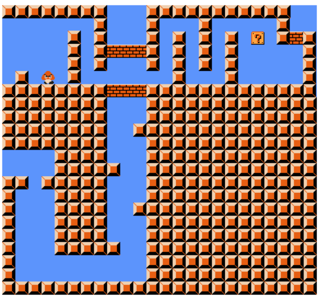


Figure 12: Crossover gadget for Super Mario Bros.

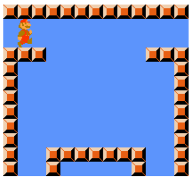


Figure 10: Variable gadget for Super Mario Bros.

shes until it is collected by Mario.

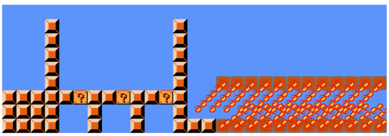
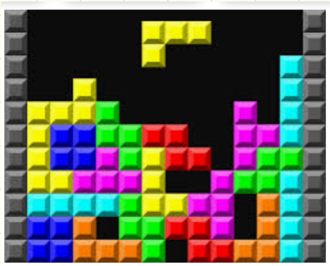


Figure 11: Clause gadget for Super Mario Bros.

Another: Tetris



NP-Hard:

Reduce 3-partition

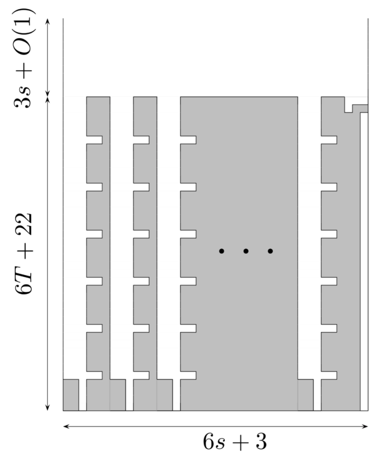


Fig. 2. *The initial gameboard for a Tetris game mapped from an instance of* 3-PARTITION.

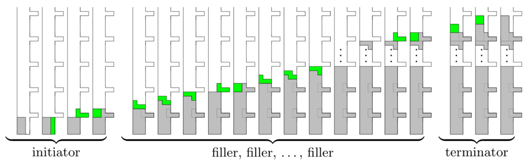


Fig. 3. *A valid sequence of moves within a bucket.*

Again: An active area of research!

arXiv.org > cs > arXiv:1711.00788

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On the complexity of optimal homotopies

Erin Wolf Chambers, Arnaud de Mesmay, Tim Ophelders

(Submitted on 2 Nov 2017)

In this article, we provide new structural results and algorithms for the Homotopy Height problem. In broad terms, this problem quantifies how much a curve on a surface needs to be stretched to sweep continuously between two positions. More precisely, given two homotopic curves γ_1 and γ_2 on a combinatorial (say, triangulated) surface, we investigate the problem of computing a homotopy between γ_1 and γ_2 where the length of the longest intermediate curve is minimized. Such optimal homotopies are relevant for a wide range of purposes, from very theoretical questions in quantitative homotopy theory to more practical applications such as similarity measures on meshes and graph searching problems.
We prove that Homotopy Height is in the complexity class NP, and the corresponding exponential algorithm is the best one known for this problem. This result builds on a structural theorem on monotonicity of optimal homotopies, which is proved in a companion paper. Then we show that this problem encompasses the Homotopic Fréchet distance problem which we therefore also establish to be in NP, answering a question which has previously been considered in several different settings. We also provide an O(log n)-approximation algorithm for Homotopy Height on surfaces by adapting an earlier algorithm of Har-Peled, Nayyeri, Salvatipour and Sidiropoulos in the planar setting.

For after break:

- Reading due by Monday
  Suggestion: Do it earlier!
  (Particularly 12.14)

- HW: due next Wednesday,
  over these reductions

- Final topic:
  Linear programming

  (Sadly, skipping approximation & randomness)