

# And... not done!

Technique	Direct	With dynamic trees	Source(s)
Blocking flow	$O(V^2E)$	$O(VE \log V)$	[Dinitz; Karzanov; Even and Itai; Sleator and Tarjan]
Network simplex	$O(V^2E)$	$O(VE \log V)$	[Dantzig; Goldfarb and Hao; Goldberg, Grigoriadis, and Tarjan]
Push-relabel (generic)	$O(V^2E)$	–	[Goldberg and Tarjan]
Push-relabel (FIFO)	$O(V^3)$	$O(VE \log(V^2/E))$	[Goldberg and Tarjan]
Push-relabel (highest label)	$O(V^2\sqrt{E})$	–	[Cheriyani and Maheshwari; Tunçel]
Push-relabel-add games	–	$O(VE \log_{E/(V \log V)} V)$	[Cheriyani and Hagerup; King, Rao, and Tarjan]
Pseudoflow	$O(V^2E)$	$O(VE \log V)$	[Hochbaum]
Pseudoflow (highest label)	$O(V^3)$	$O(VE \log(V^2/E))$	[Hochbaum and Orlin]
Incremental BFS	$O(V^2E)$	$O(VE \log(V^2/E))$	[Goldberg, Held, Kaplan, Tarjan, and Werneck]
Compact networks	–	$O(VE)$	[Orlin]

Figure 10.10. Several purely combinatorial maximum-flow algorithms and their running times.

# Even more known about "special" graphs:

## Computer Science > Data Structures and Algorithms

### Minimum Cuts in Surface Graphs

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We describe algorithms to efficiently compute minimum  $(s, t)$ -cuts and global minimum cuts of undirected surface-embedded graphs. Given an edge-weighted undirected graph  $G$  with  $n$  vertices embedded on an orientable surface of genus  $g$ , our algorithms can solve either problem in  $g^{O(g)}n \log \log n$  or  $2^{O(g)}n \log n$  time, whichever is better. When  $g$  is a constant, our  $g^{O(g)}n \log \log n$  time algorithms match the best running times known for computing minimum cuts in planar graphs. Our algorithms for minimum cuts rely on reductions to the problem of finding a minimum-weight subgraph in a given  $\mathbb{Z}_2$ -homology class, and we give efficient algorithms for this latter problem as well. If  $G$  is embedded on a surface with  $b$  boundary components, these algorithms run in  $(g + b)^{O(g+b)}n \log \log n$  and  $2^{O(g+b)}n \log n$  time. We also prove that finding a minimum-weight subgraph homologous to a single input cycle is NP-hard, showing it is likely impossible to improve upon the exponential dependencies on  $g$  for this latter problem.