CSCI 3100: Algorithms
Final Worksheet

Note: None of these problems are to be submitted. One of them will appear on the final exam.

Problems

1. The owner of a local pub, Moe, is deciding how much beer to order from the Duff Company. There are 3 types, Regular Duff, Duff Strong, and Extreme Duff. Regular Duff costs Moe \$1.50 per pint and he sells it at \$2.50 per pint; Duff Strong costs \$1.75 and he can sell it for \$3.25 per pint; and Extreme Duff costs Moe \$2.50 per pint and he can sell it for \$4.50.

   However, as part of a complex marketing scam, the Duff company will only sell a pint of Extreme Duff for every two or more pints of Regular Duff that Moe buys. Additionally, the Regular Duff and Duff Strong come in larger bottles, so Moe can only store a total of 800 of the two types combined. Finally, due to past events (better left untold), Duff will not sell Moe more than 3000 pints per week total. Moe knows that he can sell however much beer he has.

   (a) Formulate the linear program for deciding how much Regular Duff and how much Duff Strong to buy, so as to maximize Moe's profit. Note: You don't need to solve it, just set it up!
   (b) Now put the linear program into canonical form. Again, no need to solve.
   (c) Finally, construct the dual linear program. (In case I need to say it again, don't solve this part either.)

2. Give an example of a linear program in two variables whose feasible region is infinite, but such that there is an optimum solution of bounded cost.

3. As mentioned in class, an *integer program* is a linear program with the additional constraint that the variables must take on only integer values (not any real number).

   Prove that finding the optimal feasible solution to an integer program is NP-Hard.

   Hint: Almost ANY NP-Hard decision problem can be formulated as an integer program. Pick your favorite, and reduce to show hardness!

4. There are many common variations of the maximum flow problem. Here are four of them.

   (a) There are many sources and many sinks, and we wish to maximize the total flow from all sources to all sinks.
   (b) Each vertex also has a capacity on the maximum flow that can enter it.
   (c) Each edge has not only a capacity, but also a lower bound on the flow it must carry.
   (d) The outgoing flow from each node u is not the same as the incoming flow, but is smaller by a factor of $(1 - u)$, where u is a loss coefficient associated with node u.

   Each of these can be solved efficiently. Show this by reducing (a) and (b) to the original max-flow problem, and reducing (c) and (d) to linear programming.