CS3100: Algorithms
Maximum flows and minimum cuts
In class, October 25

Problems

1. Consider the following flow network where an s-t flow has been computed. The fraction next to each edge indicates the flow value (in the numerator) and the capacity (in the denominator).

   (a) What is the value of this flow? Is it a maximum s-t flow?

   (b) Find a minimum s-t cut in the flow network above. What is its capacity?

2. A new professor, teaching flows for the first time, suggests the following greedy modification to the generic Ford-Fulkerson augmenting path algorithm. Instead of using the residual graph, just reduce the capacity of any edges in the augmenting path! In particular, whenever an edge is saturated, simply remove it from the graph, and continue to find augmenting paths in the updated graphs. (If an edge is not saturated, then reset its capacity to be $c(e) - f(e)$, where $c(e)$ is the original capacity and $f(e)$ is the value of the flow on that edge.)

   Show that this algorithm does not always compute a maximum flow.

3. The University has hired you to write an algorithm to schedule final exams. Each semester, the school offers n different classes. There are r different rooms on campus and t different time slots in which exams can be offered. You are given two arrays $E[1..n]$ and $S[1..r]$, where $E[i]$ is the number of students enrolled in the i^{th} class, and $S[j]$ is the number of seats in the j^{th} room. At most one final exam can be held in each room during each time slot. Class i can hold its final exam in room j only if $E[i] < S[j]$. Describe and analyze an efficienta lgorithm to assign a room and a time slot to each class (or report correctly that no such assignment is possible).