CS3100: Algorithms
Approximation
In class, November 17

Problems

1. Recall the makespan problem discussed in class. We discussed the fact that our greedy approximation algorithm does not always give an optimal makespan assignment, but only a 2-approximation. Given an example of a set of jobs (along with a number of machines) where the greedy algorithm fails to return a solution with optimal size.

2. Recall the shortest first greedy algorithm for the interval scheduling problem that we discussed in class: Given a set of intervals, repeatedly pick the shortest interval I, delete all other intervals that overlap I, and repeat as long as there is an interval still in the set.

   In an earlier lecture, we saw that this does NOT always produce a maximum size set of non-overlapping intervals. However, it turns out to have the following interesting approximation guarantee. If s^* is the maximum size of a set of non-overlapping intervals, and s is the size of the set produced by our greedy shortest first algorithm, then $s \geq \frac{1}{2}s^*$, so that this greedy algorithm is a 2-approximation. Prove this fact.

3. Consider a different heuristic for constructing a vertex cover of a connected graph G: compute a depth first spanning tree of G, and return the set of non-leaf nodes in the tree. Prove that this set of vertices indeed is a vertex cover, and that it is in fact a 2-approximation.