CS3100

More reductions

Announcements

- HW due Wednesday

For HW: Show A is NP-Complete

① in NP - "yes" solution can be checked in poly time

② NP-Hard: reduce known NP-Hard problem to A

In general: may use any listed problems in lecture notes

Last time:

Graph reductions:

- Ind. Set
- Clique
- Vertex Cover

In lecture notes:

- Hamiltonian cycle
- Traveling salesman
  - ↳ weighted graph, design a tour that visits every vertex of minimum length

Subset Sum:

Given a set of numbers $X = \{x_1, x_2, x_3, \ldots, x_n\}$ and a target t, does some subset of X sum to t?

Ex: (actually did this one! see lecture on Sept 8)

Runtime: exponential in n

Subset Sum is NP-Hard.

Reduction: Vertex Cover

Input: Graph G & size k.

Construct a set of numbers: X

Label each edge $0, \ldots, m-1$

Add a number to X for each e_i: $4^i = b_i$

Add a number to X for each vertex v:

$$a_v : 4^m + \sum_{e_i \text{ adjacent to } v} 4^i$$

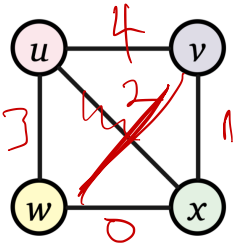
View these as base 4 #s:

edges e_i: $\underbrace{0}_{m}\ \underbrace{0 \cdots 0}_{m-1}\ \underbrace{1}_{i}\ \underbrace{0 \cdots 0}_{i}$

v: $\underbrace{1}_{m}$ ← 2's for adj edges

reduction cont:

Ex:



$a_u := 111000_4 = 1344$
$a_v := 110110_4 = 1300$
$a_w := 101101_4 = 1105$
$a_x := 100011_4 = 1029$

$b_{uv} := 010000_4 = 256$
$b_{uw} := 001000_4 = 64$
$b_{vw} := 000100_4 = 16$
$b_{vx} := 000010_4 = 4$
$b_{wx} := 000001_4 = 1$

Nice feature: for $i < m$, only 3 1's anywhere in X
$\Rightarrow$ no carrying for any subset of these!

Set $t = k \cdot 4^m + \sum_{i=0}^{m-1} 2 \cdot 4^i$

Poly time conversion:
$n + m$ numbers

reduct. cont.

$\Rightarrow$: if cover of size k $\Rightarrow$ subset of value t

Take a vertex cover in G.

For each v, choose a_v is subset of X.

have $\geq k \cdot 4^m + \sum_{i=0}^{m-1} 1 \cdot 4^i$

For every edge w/ only 1 endpoint in cover, also add b_i to subset.

$\Rightarrow$ Sum is exactly $k \cdot 4^m + \sum 2 \cdot 4^i$

$\Leftarrow$: Spps subset $X' \subseteq X$ sums to $t = k \cdot 4^m + \sum_{i=0}^{m-1} 2 \cdot 4^i$

Know close exactly k of a_v's, since lower terms can't carry.

These are a cover, since including b_i isn't enough to hit $2 \cdot 4^i$ ∎

Another: Partition

Given a set of n numbers, can you partition into 2 sets X + Y so that $\sum_{x \in X} x = \sum_{y \in Y} y$?

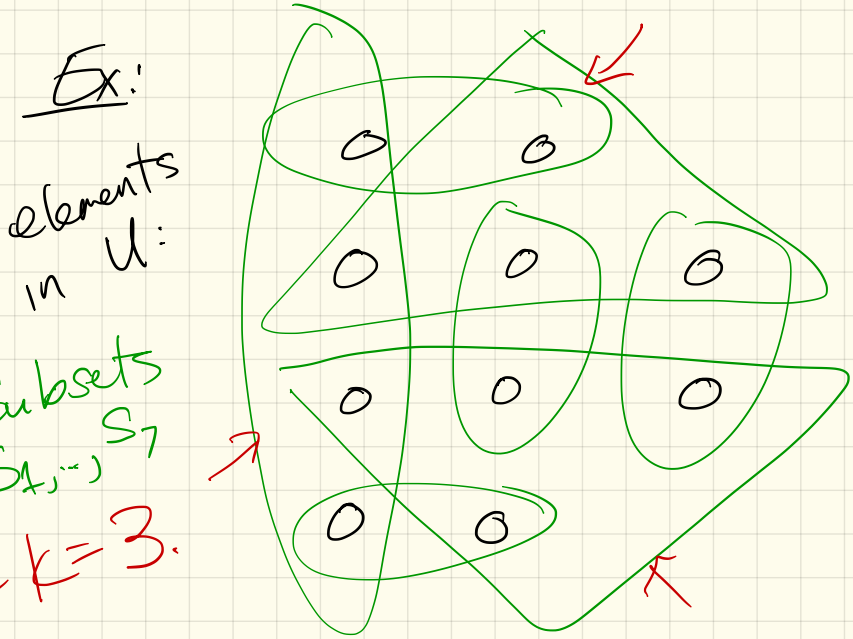
Easy reduction:

on worksheet...

Set Cover:

Given a set U of n elements, a collection $S_1, S_2, \ldots, S_m$ of subsets of U, & a number k, is there a collection of k of the S_i's whose union is all of U?

Ex:



Answer? Yes!

Set Cover is NP-Hard:

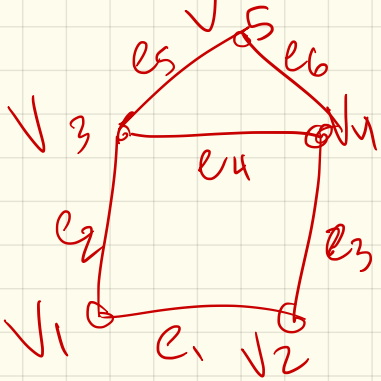
Reduction from vertex cover, so input is G & k.

Construct:

U = edges = $\{e_1, \ldots, e_m\}$

S_i's: S_i = {edges adjacent to vertex i}

& k: k (same)



$U = \{e_1 \ldots e_6\}$

$S_1 = \{e_1, e_2\}$

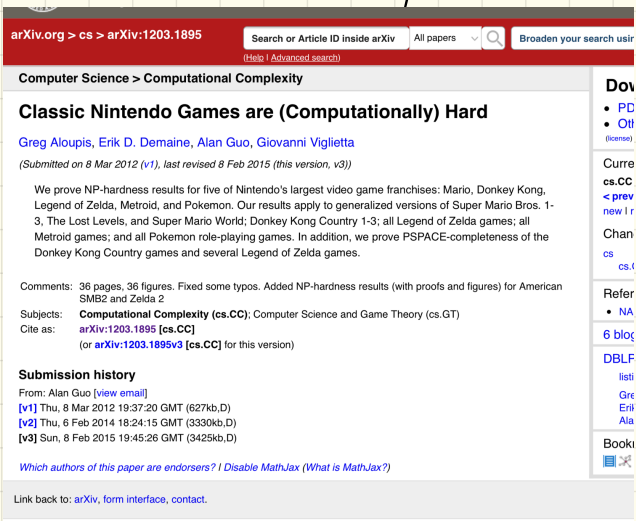
$S_2 = \{e_1, e_3\}$

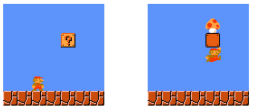
$\vdots$

Vertex cover of size k

$\Longleftrightarrow$ set cover of size k

Some fun examples





Left: Start gadget for Super Mario Bros. Right: The item block contains a

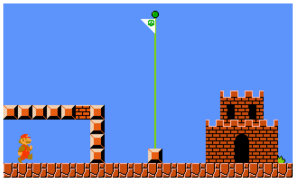


Figure 9: Finish gadget for Super Mario Bros.

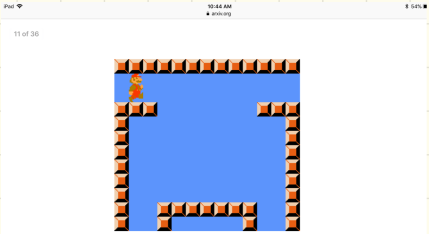


Figure 10: Variable gadget for Super Mario Bros.

shes until it is collected by Mario.

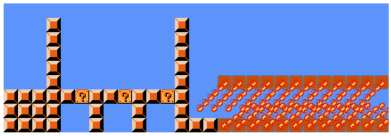


Figure 11: Clause gadget for Super Mario Bros.

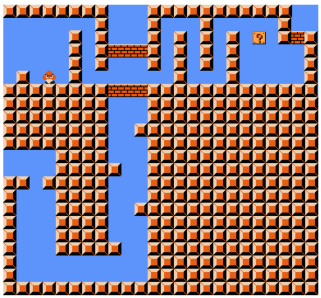
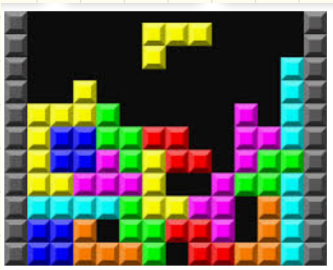


Figure 12: Crossover gadget for Super Mario Bros.

Another: Tetris



NP-Hard:

Reduce 3-partition

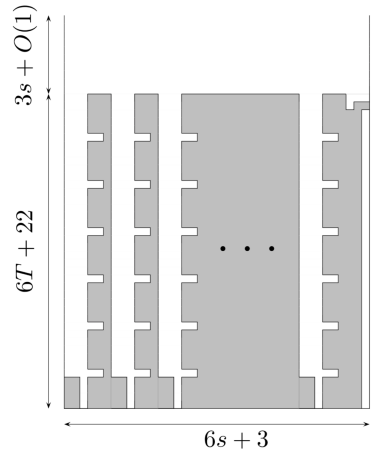


Fig. 2. *The initial gameboard for a Tetris game mapped from an instance of* 3-PARTITION.

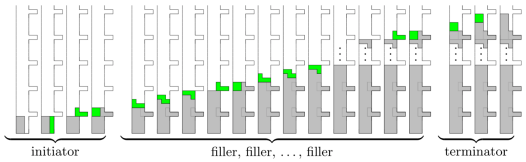


Fig. 3. *A valid sequence of moves within a bucket.*

Next time: