

CS3/00

More reductions



Announcements

- HW due Wednesday

For HW: Show A is NP-Complete

① in NP - "yes" solution can be checked in poly time

② NP-Hard is reduce known NP-Hard problem to A

In general: may use any listed problems in lecture notes

Last time :

Graph reductions:

- Ind. Set
- Clique
- Vertex Cover

In lecture notes:

- Hamiltonian cycle
- Traveling Salesman

↳ weighted graph, design a tour that visits every vertex of minimum length

Subset Sum :

Given a set of numbers

$X = \{x_1, x_2, x_3, \dots, x_n\}$
and a target t , does
some subset of X sum to t ?

Ex: (actually did this one!
see lecture on Sept 8)

Runtime: exponential
in n

Subset Sum is NP-Hard.

Reduction: Vertex Cover

Input: Graph G & size k .

Construct a set of numbers: X

Label each edge $0, \dots, m-1$

Add a number to X for each e_i : $4^i = b_i$

Add a number to X for each vertex v :

$$a_v = 4^m + \sum_{\substack{e_i \text{ adjacent} \\ \text{to } v}} 4^i$$

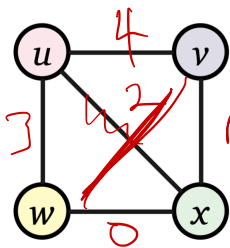
View these as base 4 #s:

$$\text{edges } e_i: \underbrace{0}_{m} \underbrace{0 \dots 0}_{m-1} \underbrace{1}_i \dots \underbrace{0}_0$$

$$v: \underbrace{1}_m \quad \leftarrow \text{for edges adj}$$

reduction cont:

Ex:



$$a_u := 111000_4 = 1344$$

$$a_v := 110110_4 = 1300$$

$$a_w := 101101_4 = 1105$$

$$a_x := 100011_4 = 1029$$

$$b_{uv} := 010000_4 = 256$$

$$b_{uw} := 001000_4 = 64$$

$$b_{vw} := 000100_4 = 16$$

$$b_{vx} := 000010_4 = 4$$

$$b_{wx} := 000001_4 = 1$$

Nice feature: for $i < m$,
only 3 1's anywhere in X
 \Rightarrow no carrying for any
subset of these!

$$\text{Set } t = k \cdot 4^m + \sum_{i=0}^{m-1} 2 \cdot 4^i$$

Poly time conversion:
 $n + m$ numbers

⇒: reduct. cont.

if cover of size k
⇒ subset of value t

Take a vertex cover in G .

For each v , choose a_v
is subset of X .

$$\text{have } \geq k \cdot 4^m + \underbrace{\sum_{i=0}^{m-1} 1 \cdot 4^i}$$

For every edge w of only 1
endpoint in cover, also
add b_i to subset.

$$\Rightarrow \text{Sum is exactly } k \cdot 4^m + \sum 2 \cdot 4^i$$

⇐: Spns subset $X' \subseteq X$
sums to $t = k \cdot 4^m + \sum_{i=0}^{m-1} 2 \cdot 4^i$

Know chose exactly k of a_v 's,
since lower terms can't carry.

These are a cover, since including
 b_i isn't enough to hit $2 \cdot 4^i$ □

Another: Partition

Given a set of n numbers,

can you partition into 2 sets X + Y so that

$$\sum_{x \in X} x = \sum_{y \in Y} y \quad ?$$

Easy reduction:

on worksheet...

Set Cover:

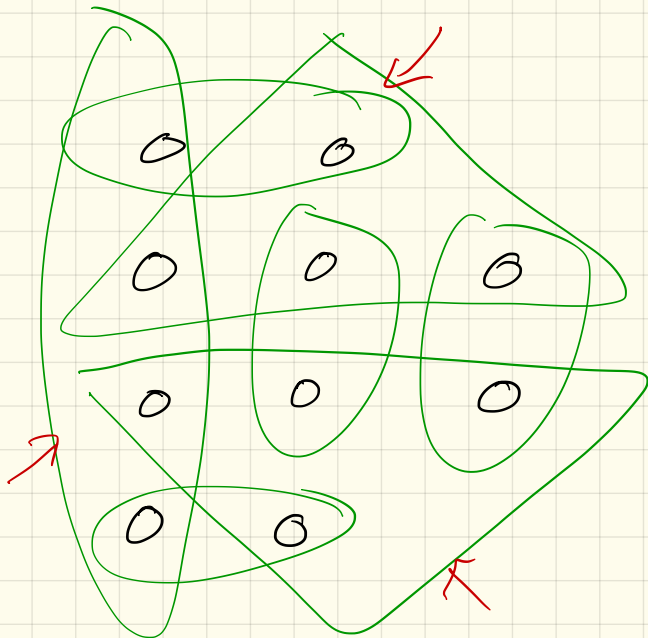
Given a set U of n elements,
- a collection S_1, S_2, \dots, S_m of
subsets of U , & a number k ,
is there a collection of k
of the S_i 's whose union is
all of U ?

Ex:

elements
in U :

Subsets
 S_1, \dots, S_7

& $k=3$.



Answer? Yes!

Set Cover is NP-Hard:

Reduction from vertex cover,

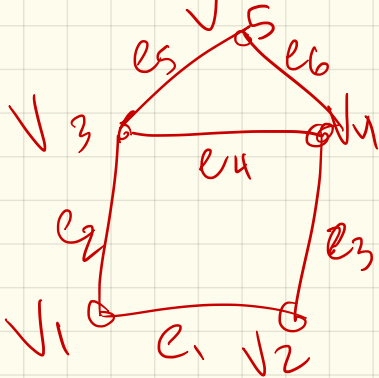
so input is G & k .

Construct:

$$U = \text{edges} = \{e_1, \dots, e_m\}$$

$$S_i \text{'s} : S_i = \{ \text{edges adjacent to vertex } v_i \}$$

v & k : k (same)



$$U = \{e_1, \dots, e_6\}$$
$$S_1 = \{e_1, e_2\}$$
$$S_2 = \{e_1, e_3\}$$
$$\vdots$$

Vertex cover of size k
 \Leftrightarrow set cover
of size k

Some fun examples

arXiv.org > cs > arXiv:1203.1895

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Computer Science > Computational Complexity

Classic Nintendo Games are (Computationally) Hard

Greg Aloupis, Erik D. Demaine, Alan Guo, Giovanni Viglietta

(Submitted on 8 Mar 2012 (v1), last revised 8 Feb 2015 (this version, v3))

We prove NP-hardness results for five of Nintendo's largest video game franchises: Mario, Donkey Kong, Legend of Zelda, Metroid, and Pokemon. Our results apply to generalized versions of Super Mario Bros. 1-3, The Lost Levels, and Super Mario World; Donkey Kong Country 1-3; all Legend of Zelda games; all Metroid games; and all Pokemon role-playing games. In addition, we prove PSPACE-completeness of the Donkey Kong Country games and several Legend of Zelda games.

Comments: 36 pages, 36 figures. Fixed some typos. Added NP-hardness results (with proofs and figures) for American SMB2 and Zelda 2

Subjects: **Computational Complexity (cs.CC)**; Computer Science and Game Theory (cs.GT)

Cite as: arXiv:1203.1895 [cs.CC] (or arXiv:1203.1895v3 [cs.CC] for this version)

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From: Alan Guo [view email]

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[v3] Sun, 8 Feb 2015 19:45:26 GMT (3425kb,D)

Which authors of this paper are endorsers? | Disable MathJax (What is MathJax?)

Link back to: arXiv, form interface, contact.

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Current version: cs.CC < prev new | r

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- NA

6 blog

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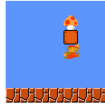
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Bookmarks



Left: Start gadget for Super Mario Bros. Right: The item block contains a

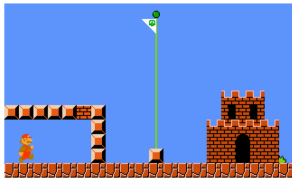


Figure 9: Finish gadget for Super Mario Bros.

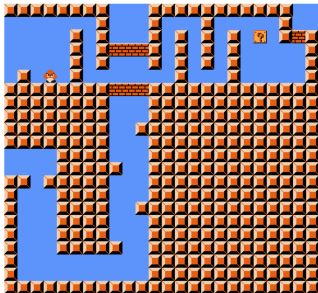


Figure 12: Crossover gadget for Super Mario Bros.



Figure 10: Variable gadget for Super Mario Bros.

shes until it is collected by Mario.

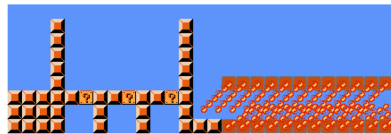
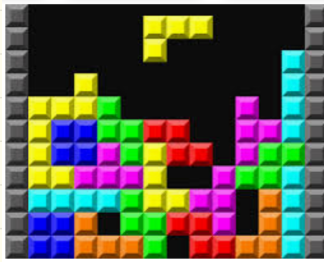


Figure 11: Clause gadget for Super Mario Bros.

Another: Tetris



NP-Hard:
Reduce 3-partition

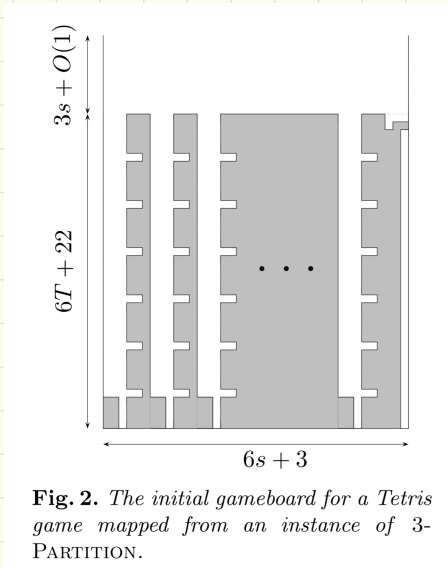
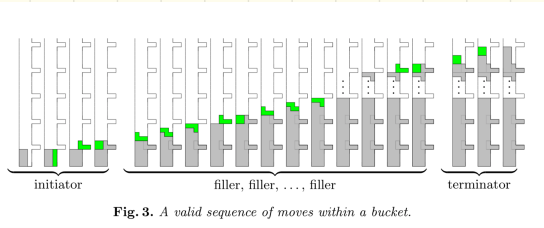


Fig. 2. The initial gameboard for a Tetris game mapped from an instance of 3-PARTITION.



Next time: