CS3100: Algorithms
Homework 9

1. The owner of a local pub, Moe, is deciding how much beer to order from the Duff Company. There are 3 types, Regular Duff, Duff Strong, and Extreme Duff. Regular Duff costs Moe $1.50 per pint and he sells it at $2.50 per pint; Duff Strong costs $1.75 and he can sell it for $3.25 per pint; and Extreme Duff costs Moe $2.50 per pint and he can sell it for $4.50.

   However, as part of a complex marketing scam, the Duff company will only sell a pint of Extreme Duff for every two or more pints of Regular Duff that Moe buys. Additionally, the Regular Duff and Duff Strong come in larger bottles, so Moe can only store a total of 800 of the two types combined. Finally, due to past events (better left untold), Duff will not sell Moe more than 3000 pints per week total. Moe knows that he can sell however much beer he has.

   (a) Formulate the linear program for deciding how much Regular Duff and how much Duff Strong to buy, so as to maximize Moe's profit. Note: You don't need to solve it, just set it up!

   (b) Now put the linear program into canonical form. Again, no need to solve.

   (c) Finally, construct the dual linear program. (In case I need to say it again, don't solve this part either.)

2. For this problem, we define a salad as any combination of the following ingredients: (1) tomato, (2) lettuce, (3) spinach, (4) carrot, and (5) ranch. Each valid salad must contain: (a) at least 25 grams of protein, (b) at least 2 and at most 10 grams of fat, (c) at least 25 grams of carbohydrates, (d) at most 100 milligrams of sodium. Furthermore, (e) you do not want your salad to be more than 70% greens by mass (measured in grams of each ingredient, where lettuce and spinach both count as greens), and (f), at least 5% of mass must be ranch dressing (because it is tasty).

   The nutritional contents of these ingredients per 100 grams are:

ingredient	energy (kcal)	protein (grams)	fat (grams)	carbs (grams)	sodium (milligrams)
tomato	18	.9	0.2	3.9	9
lettuce	15	1.4	0.20	2.9	8
spinach	23	2.9	.4	3.6	7
carrot	41	.9	.2	10	10
ranch	484	1	100.00	7.5	1094

   Find a linear programming applet on the web (there are a ton of them!) and use it to make the salad with the fewest calories under the nutritional constraints described above. Describe your linear programming formula and the optimal solution (both the quantity of each ingredient and the total value). Please make sure to cite the page you use, and include a screenshot or printout of the setup/solution you used. (If you are presenting orally, you'll describe to me the LP you formulated, and you can just bring a screen shot or printout of the tool you used.)

3. As mentioned in class, an *integer program* is a linear program with the additional constraint that the variables must take on only integer values (not any real number).

   Prove that finding the optimal feasible solution to an integer program is NP-Hard.

   Hint: Almost ANY NP-Hard decision problem can be formulated as an integer program. Pick your favorite, and reduce to show hardness!