CS3100: Algorithms

Homework 6:
More flow and reductions

1. Suppose you are running a web site that is visited by the same set of people every day. Each visitor claims membership in one or more demographic groups; for example, a visitor might describe herself as female, 30–40 years old, a mother, a resident of Missouri, an academic, and a fan of Joss Whedon. Your site is supported by advertisers. Each advertiser has told you which demographic groups should see its ads and how many of its ads you must show each day. Altogether, there are n visitors, k demographic groups, and m advertisers.

   Describe an efficient algorithm to determine, given all the data described in the previous paragraph, whether you can show each visitor exactly one ad per day, so that every advertiser has its desired number of ads displayed, and every ad is seen by someone in an appropriate demographic group.

2. A boolean formula is in disjunctive normal form (or DNF) if it consists of a disjunction (OR) or several terms, each of which is the conjunction (AND) of one or more literals. For example, the formula:

   $$(\overline{x} \wedge y \wedge \overline{z}) \vee (y \wedge z) \vee (x \wedge \overline{y} \wedge \overline{z})$$

   is in disjunctive normal form. DNF-SAT asks, given a boolean formula in disjunctive normal form, whether that formula is satisable.

   (a) Describe a polynomial-time algorithm to solve DNF-SAT.

   (b) What is the error in the following argument that P=NP?

   Suppose we are given a boolean formula in conjunctive normal form with at most three literals per clause, and we want to know if it is satisable. We can use the distributive law to construct an equivalent formula in disjunctive normal form. For example,

   $$(x \vee y \vee \overline{z}) \wedge (\overline{x} \vee \overline{y}) \iff (x \wedge \overline{y}) \vee (y \wedge \overline{x}) \vee (\overline{z} \wedge \overline{x}) \vee (\overline{z} \wedge \overline{y})$$

   Now we can use the algorithm from part (a) to determine, in polynomial time, whether the resulting DNF formula is satisable. We have just solved 3SAT in polynomial time. Since 3SAT is NP-hard, we must conclude that P=NP!

3. Consider the following problem: Given as input two graphs H and G, is H isomorphic to to some subgraph of G? Prove that this problem is NP-Complete.

   (Hint: remember you need to reduce some known NP-Hard problem to this one, and consider the graph problems we have seen in class for your reduction.)