CS3100: Algorithms
Homework 4

1. Give an algorithm to detect whether an undirected graph contains a cycle. If the graph contains a cycle, your algorithm should output one. (Note: not all of them - just one!) The running time of your algorithm should be $O(m+n)$ for a graph with n nodes and m edges.

   Hint: You are welcome to modify and/or use one of the graph algorithms from class to get pseudocode, but be sure to be precise in exactly how you are modifying the data structures and pseudocode that your algorithm is based on.

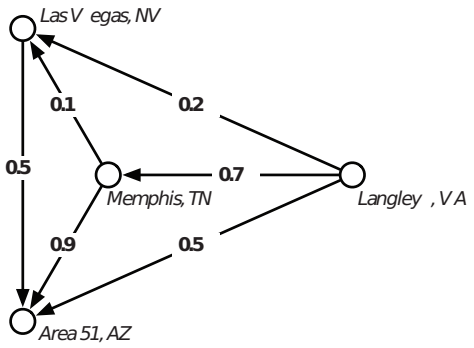
2. Let $G = (V, E)$ be an undirected graph with a weight $w(e)$ for each edge $e \in E$. Assume you are given a minimum spanning tree T for G. Now assume that a single new edge e is added to G, connecting two nodes u and v and with a cost c.

   (a) Give an efficient algorithm to test if T is still a minimum spanning tree for this new graph. Make your algorithm run in $O(m+n)$ time. Can you do it in $O(n)$ time? Please be sure to note any assumptions you make about what data structure is used to represent both T and G.

   (b) Suppose T is no longer the minimum spanning tree for G. Give an algorithm that is as fast as possible to update T to the new minimum spanning tree.

3. Mulder and Scully have computed, for every road in the United States, the exact probability that someone driving on that road wont be abducted by aliens. Agent Mulder needs to drive from Langley, Virginia to Area 51, Nevada. What route should he take so that he has the least chance of being abducted?

   More formally, you are given a directed graph $G = (V, E)$ where every edge e has an independent safety probability $p(e)$. The safety of a path is the product of the safety probabilities of its edges. Design and analyze an algorithm to determine the safest path from a given start vertex s to a given target vertex t.



For example, with the probabilities shown above, if Mulder tries to drive directly from Langley to Area 51, he has a 50% chance of getting there without being abducted. If he stops in Memphis, he has a $0.7 \times 0.9 = 63\%$ chance of arriving safely. If he stops first in Memphis and then in Las Vegas, he has a $1 - 0.7 \times 0.1 \times 0.5 - 96.5\%$ chance of being abducted! (That's how they got Elvis, you know.)

4. **Extra credit:** (To be handed in on paper if you choose to do it.)

   Let G be a connected graph, and let T be a depth-first spanning tree of G rooted at some node v. Prove that if T is also a breadth-first spanning tree of G rooted at v, then $G = T$ (so G cannot contain any other edges).