CS314: Algorithms
Homework 0

Required Problems

1. Recurrences (20 points)

Solve the following recurrences. State tight asymptotic bounds for each function in the form $\Theta(f(n))$ for some recognizable function $f(n)$. You do not need to turn in proofs (in fact, please *don't* turn in proofs), but you should do them anyway just for practice. Assume reasonable but nontrivial base cases if none are supplied. More exact solutions are better.

(a) $A(n) = 2A(n/2) + \lg n$

(b) $B(n) = 3B(n/2) + n$

(c) $C(n) = 2C(n/2) + n^2$

(d) $D(n) = 3D(n-1) + 1$

(e) $E(n) = 2E(\lfloor n/3 \rfloor + 9) + n^2$

(f) $F(n) = 2F(n-1)/F(n-2)$

(g) $G(n) = G(n/2) + 1$

2. Sorting functions (20 points)

Sort the following 25 functions from asymptotically smallest to asymptotically largest, indicating ties if there are any. You do not need to turn in proofs (in fact, please *don't* turn in proofs), but you should do them anyway just for practice.

1	n	n^2	$\lg n$	$\lg \sqrt{n}$
$\cos n + 2$	$n^{\lg n}$	$\lg 2^n$	$2^{\sqrt{\lg n}}$	$\lg 2^{\sqrt{n}}$
$\sqrt{2^{\lg n}}$	$2^{\lg n}$	$n \lg n$	$\sum_{i=1}^{n} i$	$\lg \sqrt{2^n}$

To simplify notation, write $f(n) \ll g(n)$ to mean $f(n) = o(g(n))$ and $f(n) \equiv g(n)$ to mean $f(n) = \Theta(g(n))$. For example, the functions n^2, n, $\binom{n}{2}$, n^3 could be sorted either as $n \ll n^2 \equiv \binom{n}{2} \ll n^3$ or as $n \ll \binom{n}{2} \equiv n^2 \ll n^3$. *[Hint: When considering two functions $f(\cdot)$ and $g(\cdot)$ it is sometime useful to consider the functions $\ln f(\cdot)$ and $\ln g(\cdot)$.]*

3. The traditional Devonian/Cornish drinking song "The Barley Mow" has the following pseudolyrics[1], where *container*$[i]$ is the name of a container that holds 2^i ounces of beer. One version of the song uses the following containers: nipperkin, gill pot, half-pint, pint, quart, pottle, gallon, half-anker, anker, firkin, half-barrel, barrel, hogshead, pipe, well, river, and ocean. (Every container in this list is twice as big as its predecessor, except that a firkin is actually 2.25 ankers, and the last three units are just silly.)

[1] Pseudolyrics are to lyrics as pseudocode is to code.

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BarleyMow(n):
  "Here's a health to the barley-mow, my brave boys,"
  "Here's a health to the barley-mow!"

  "We'll drink it out of the jolly brown bowl,"
  "Here's a health to the barley-mow!"
  "Here's a health to the barley-mow, my brave boys,"
  "Here's a health to the barley-mow!"

  for i ← 1 to n
      "We'll drink it out of the container[i], boys,"
      "Here's a health to the barley-mow!"
      for j ← i downto 1
          "The container[j],"
      "And the jolly brown bowl!"
      "Here's a health to the barley-mow!"
      "Here's a health to the barley-mow, my brave boys,"
      "Here's a health to the barley-mow!"
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(a) Suppose each container name *container*$[i]$ is a single word, and you can sing four words a second. How long would it take you to sing BarleyMow(n)? Give the best bound you can in the form $\Theta(f(n))$ for some simple function f.

(b) If you want to sing this song for $n > 20$, you'll have to make up your own container names. To avoid repetition, these names must get progressively longer as n increases.[2] Suppose *container*$[n]$ has $\Theta(\log n)$ syllables, and you can sing six syllables per second. Now how long would it take you to sing BarleyMow(n)? Give the best bound you can in the form $\Theta(f(n))$ for some simple function f.

(c) Suppose each time you mention the name of a container, you actually drink the corresponding amount of beer: one ounce for the jolly brown bowl, and 2^i ounces for each *container*$[i]$. Assuming for purposes of this problem that you are at least 21 years old, how many ounces of beer would you drink if you sang BarleyMow(n)? Give the best bound you can in the form $\Theta(f(n))$ for some simple function f.

4. Dr. Chambers recently returned from Germany with a new favorite 24-node binary tree, in which every node is labeled with a unique letter from the German alphabet. (Note that this is pretty similar to English, but adds interesting characters like the umlaut and ß.) She gives you the following traversals:

   - Preorder: B K Ü E H L Z I Ö R C ß T S O A Ä D F M N U G
   - Postorder: H I Ö Z R L E C Ü S O T A ß K D M U G N F Ä B

   (a) List the nodes in an in order traversal of the tree.

   (b) Draw the tree.

[2] "We'll drink it out of the hemisemidemiyottapint, boys!"