CS314 – Network Flow

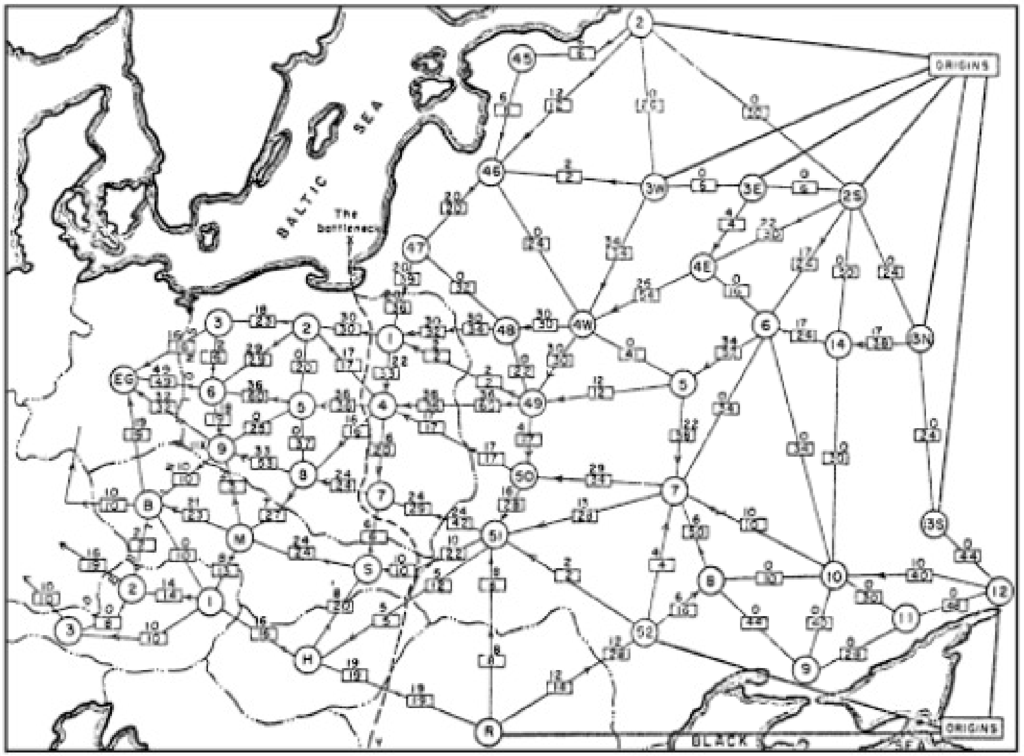
Note Title 10/23/2013

Announcements

- Midterms graded: average 35.4/60
  (don't worry, will curve!)
- Boeing scholarships
- HW due Friday
- Office hours tomorrow: 11–12

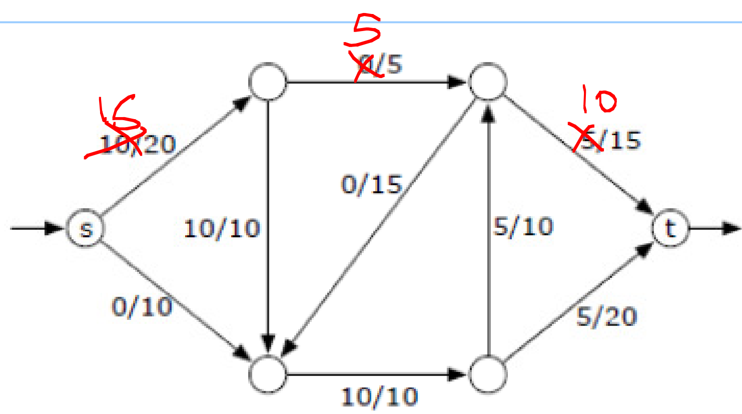
Last week : Network Flow

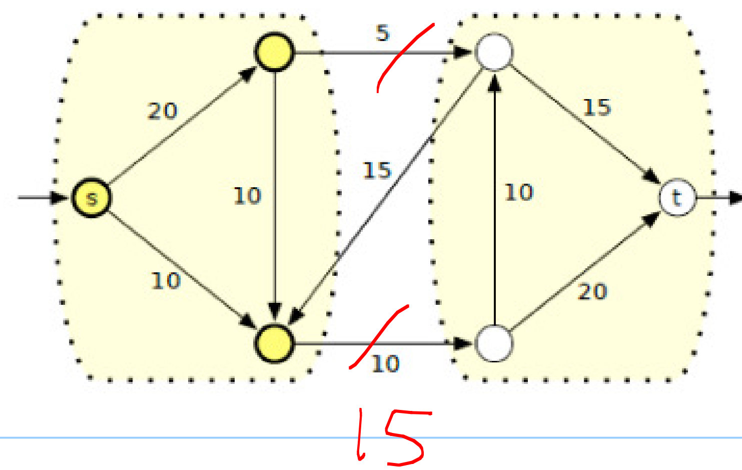
(classified report from 1950's)



Harris and Ross's map of the Warsaw Pact rail network

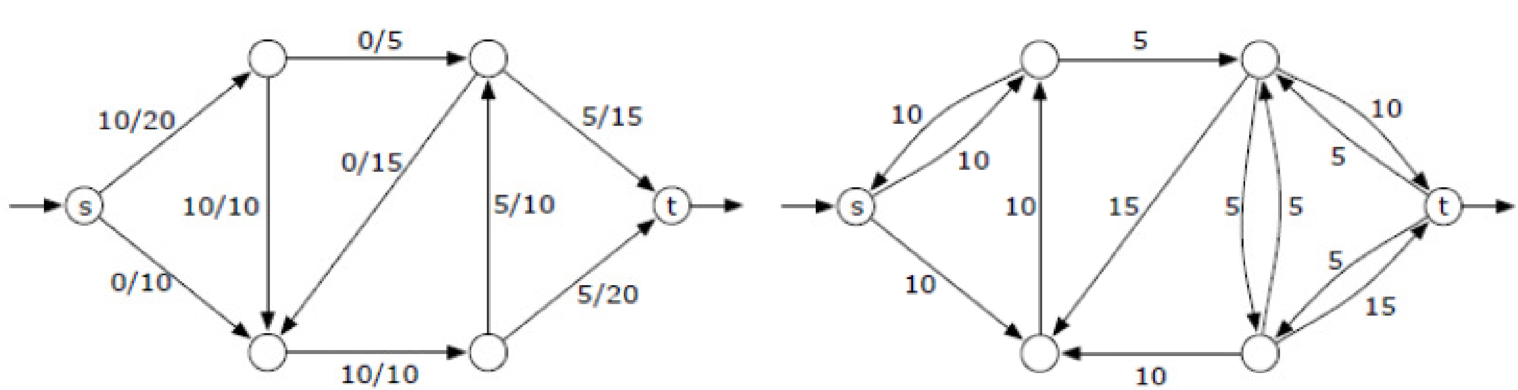
Flows vs. Cuts





Key Thm: Max flow = min cut

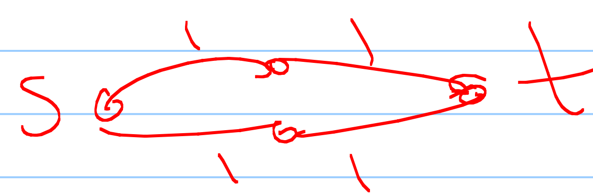
Tool: The residual graph



A flow f in a weighted graph G and the corresponding residual graph G_f.

FF: $O(m \cdot F)$

↑ value of flow



<u>Today</u>: applications

Max flow is useful in <u>many</u> settings.

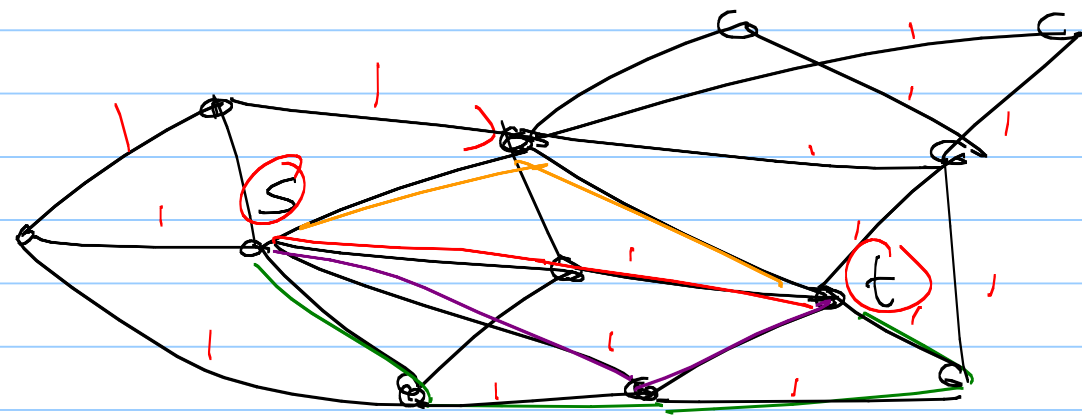
Given a problem, the main steps are:

1. Turn it into a "flow" graph
2. Plug in F-F + get a runtime
3. Correctness
   flow $\Longleftrightarrow$ solution to problem

Example: Disjoint Paths

Goal: Find the number of edge disjoint paths in a graph (between 2 vertices).

(directed)



How?

Put capacity 1 on every edge,
& run F-F.

<u>Runtime</u>:

Correctness:

k paths $\iff$ flow of size k

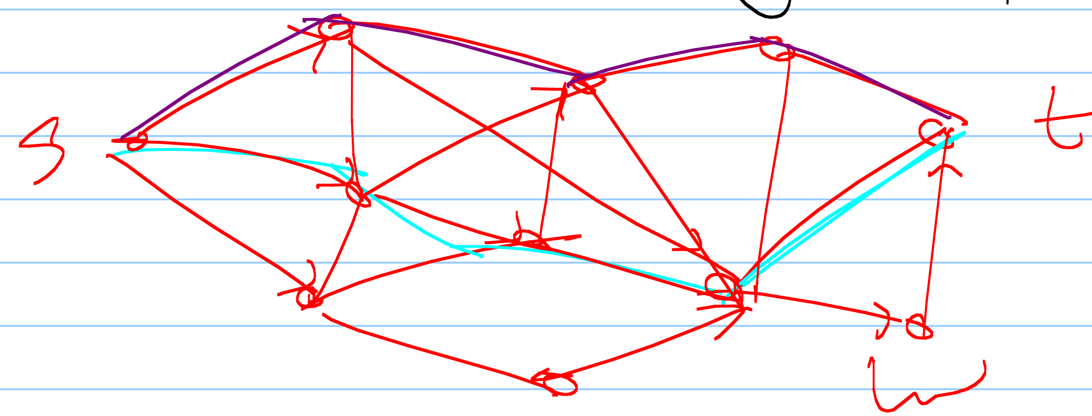
$\Rightarrow$: original G has k paths.
push 1 unit of flow on each path.

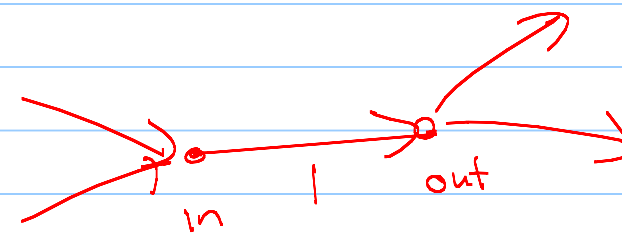
$s \xrightarrow{1} \circ \xrightarrow{1} \circ \longrightarrow t$ flow has size k

$\Leftarrow$: if flow of size k:
know flow puts 0 or 1 on each edge

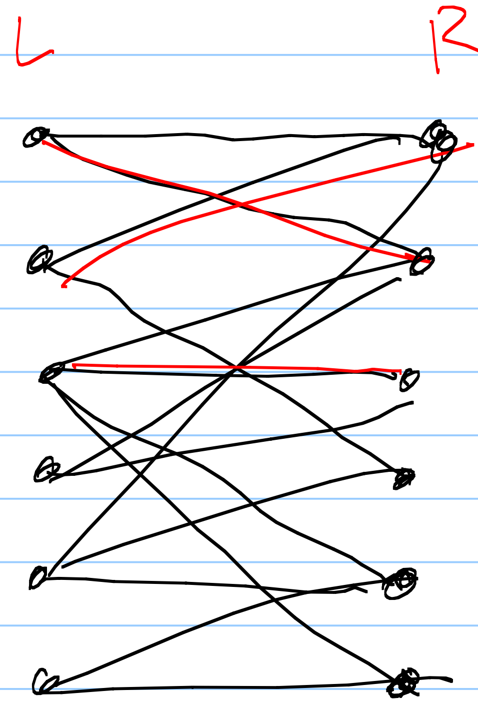
path: starting at s, follow edge of length 1.
must be another edge out of flow 1.
continue until hit t.
(repeat k times)

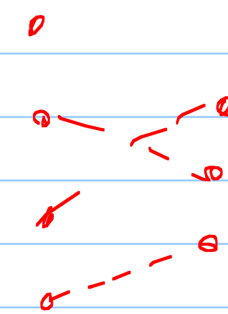
What about vertex disjoint paths?





Example: Bipartite Matching





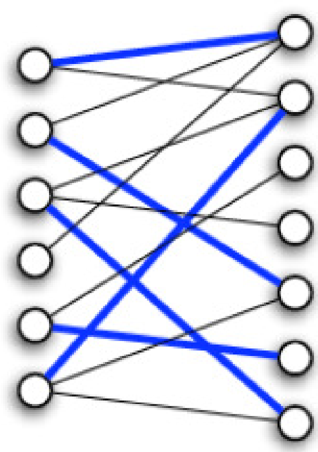
Why?

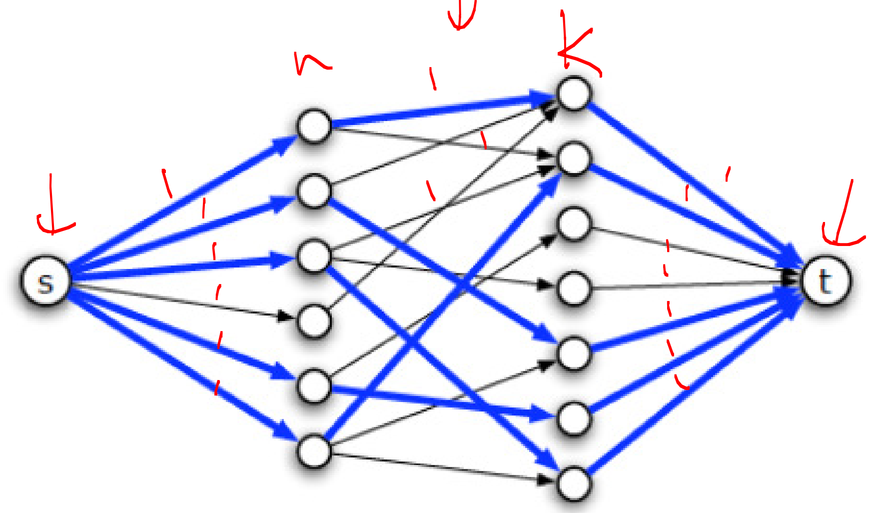
scheduling: match people to objects

etc...

How?

Flows!





capacity one on every edge

Construct a new graph:

edges $\leq nk$

Runtime:

$$O(mF) \quad (FF)$$

$$\shortparallel$$

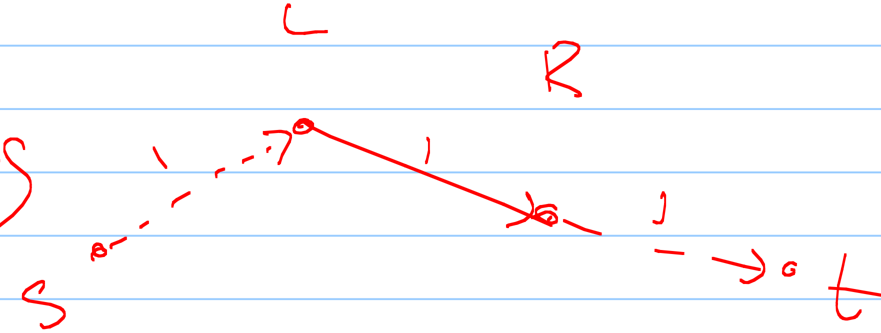
$$O(n \cdot k \cdot \min(n,k))$$

Correctness:

matching $\Longleftrightarrow$ flow

matching in original graph: for each $e = uv$ in matching, send 1 flow along $s \to u$, 1 along $u \xrightarrow{} v$, 1 along $v \to t$

do this for each edge in matching $\Rightarrow$ get a flow



Another: Assigment Problems

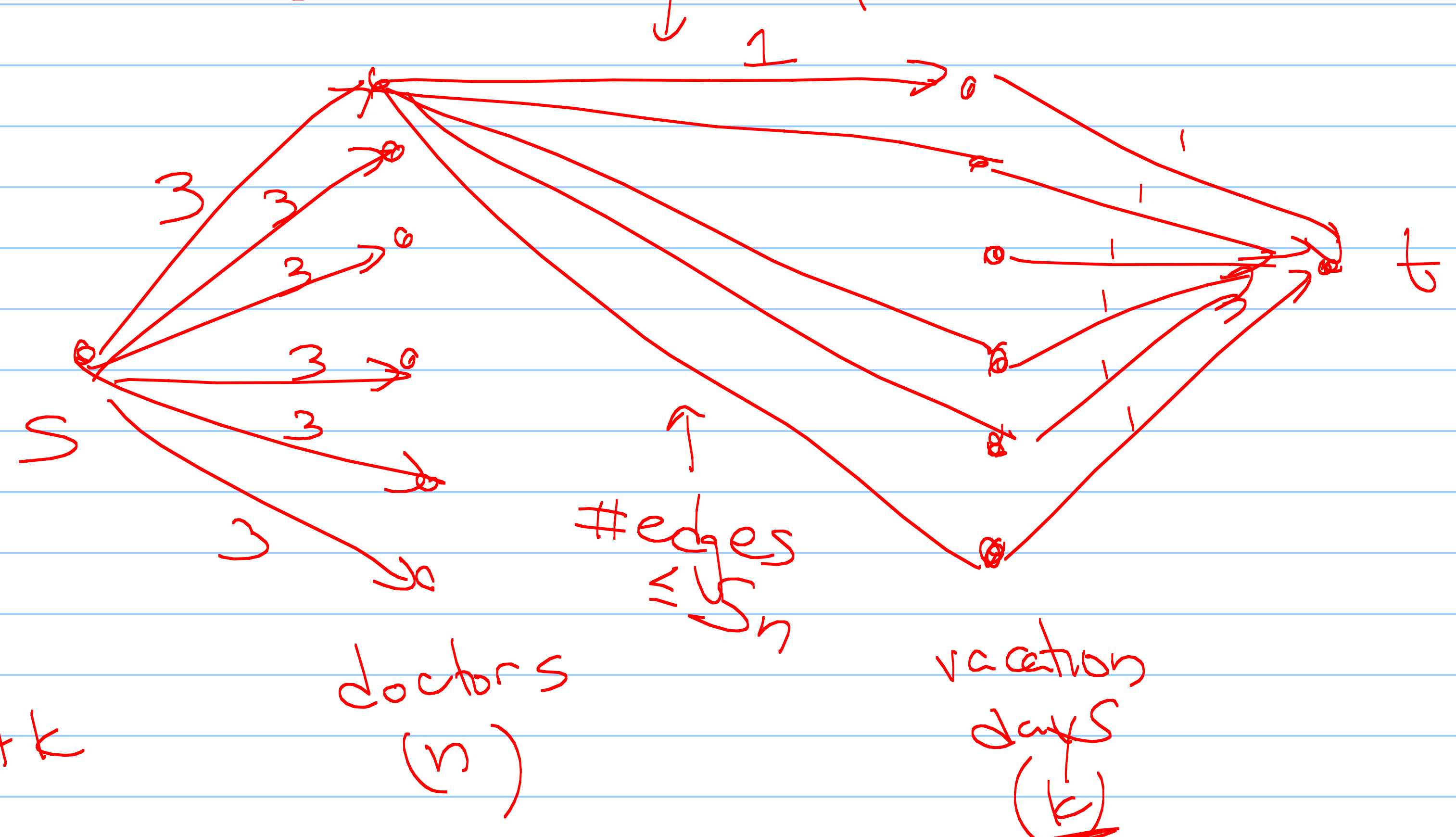
n doctors at a hospital
k vacation days

Need:
- a doctor scheduled on every vacation day
- no doctor scheduled on more than 3 vacation days
- each doctor submits a list of ≥ 5 vacation days they are available to work.

Q: is there a feasible schedule?

Q: is there a flow of size k?

Build G:



$5 \cdot 6 n + k$