

CS314: Algorithms

Graph Algorithms

In class, October 9

Problems

1. Consider a graph G that is an arbitrary undirected graph with distinct weights on the edges. Prove or disprove the following:
 - (a) The minimum spanning tree of G includes the lightest edge in every cycle in G .
 - (b) The minimum spanning tree of G excludes the heaviest edge in every cycle in G .

2. Let us say that a graph G is a near-tree if it is connected and has at most $n + 8$ edges. Give an algorithm with $O(n)$ running time that takes a near-tree G with edge weights and returns a minimum spanning tree of G . You may assume that all the edges have distinct weights.

(Hint: the previous problem might be useful.)

3. After graduating you accept a job with Aerophobes- R-Us, the leading traveling agency for people who hate to fly. Your job is to build a system to help customers plan airplane trips from one city to another. All of your customers are afraid of flying (and by extension, airports), so any trip you plan needs to be as short as possible. You know all the departure and arrival times of all the flights on the planet.

Suppose one of your customers wants to fly from city X to city Y. Describe an algorithm to find a sequence of flights that minimizes the total time in transit - the length of time from the initial departure to the final arrival, including time at intermediate airports waiting for connecting flights.

(Hint: Modify the input data and apply Dijkstra's algorithm.)