CS314 – Approximation

Note Title 11/11/2013

Announcements

- Oral grading day tomorrow
- Next HW up tomorrow, due in 1 week

Last time:

Approximating via greedy strategies:

- Makespan: scheduling n jobs to m machines
  - 2-approx
  - offline: 3/2-approx
- Vertex cover

  $O(\log n)$-approx

More on vertex cover:

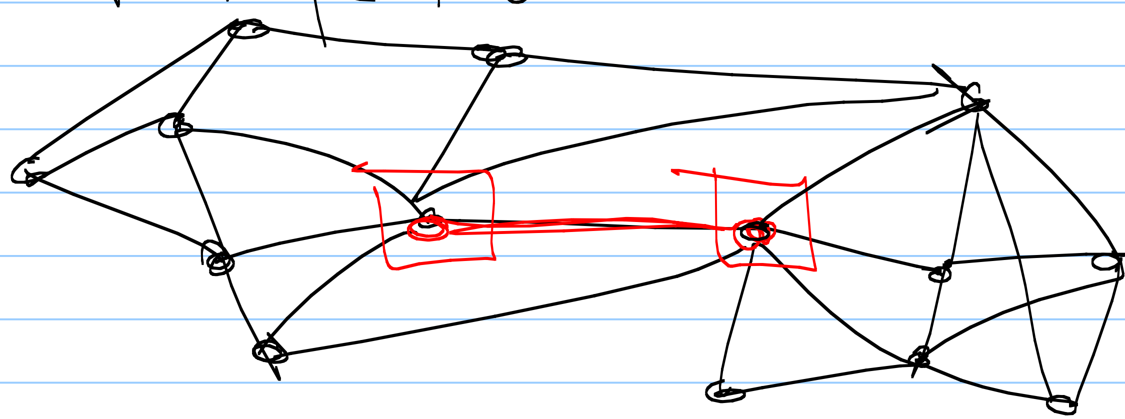
- An $O(\log n)$ approx means

$$|\text{greedy cover}| \le c \cdot \log n \cdot |OPT|$$

Sometimes greedy still isn't good!
Let's try something simpler... ☺

Simple idea:

- Pick an edge, add endpoints to the cover
- Delete all newly covered edges & repeat



$\underline{\text{DUMBVERTEXCOVER}(G)\text{:}}$

$C \leftarrow \varnothing$

while G has at least one edge

$\quad (u, v) \leftarrow$ any edge in G

$\quad G \leftarrow G \setminus \{u, v\}$

$\quad C \leftarrow C \cup \{u, v\}$

return C

Thm: Dumb vertex cover is a 2-approx.

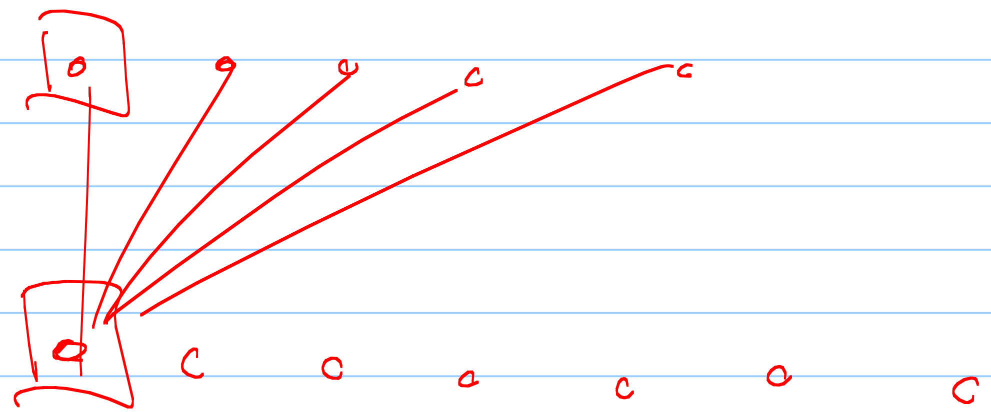
pf: Consider optimal cover C^*.

For any edge $e = \{v, w\}$, know $v \in C^*$ or $w \in C^*$.

In worst case, dumb vertex cover took v & w instead of just one.

$$\Rightarrow |C| \leq 2 \cdot |C^*|$$

$\boxtimes$



greedy is not always best!

Another: Traveling Salesman

Given n cites with pairwise distances, find shortest cycle visiting all cities.

NP-Hard: reduction from Ham cycle.

Given G, make G':

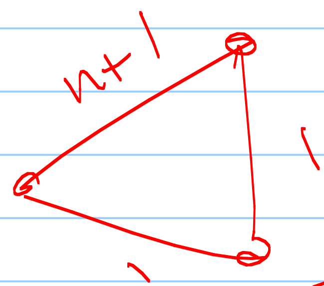
put $e \in G'$ for every v, w

$$\begin{cases} \cdot\, w(e) = 1 & \text{if } e \in G \\ w(e) = 2 & \text{if } e \notin G \end{cases}$$

Note: Nothing special about 1 & 2!

Choosing far apart values allows us to prove that even approximating TSP is NP-Hard.

Ex: Let $G' = \begin{cases} w(e) = 1 & \text{if } e \in G \\ w(e) = n+1 & \text{if } e \notin G \end{cases}$



(no $\triangle$-ineq.)

Then G has Ham cycle
$\iff$ G' has TSP tour of length n

And, if no Ham cycle, G' has tour only of length $\geq 2n$

So - if we could approx. TSP within a factor of 2, could solve Ham cycle!

Thm: For any polynomial $f(n)$, there is no $f(n)$-approx for TSP, unless $P = NP$.

So — no hope...

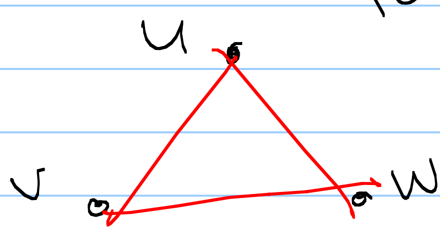
Using extra structure:

Still possible to approximate if we have extra knowledge about the graph.

Dfn: triangle inequality:

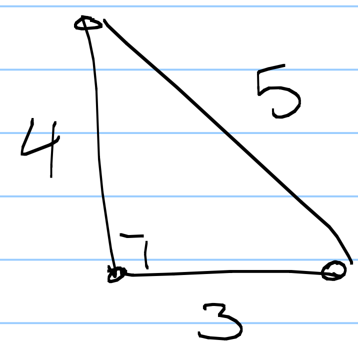
$$\ell(u,w) \le \ell(u,v) + \ell(v,w)$$

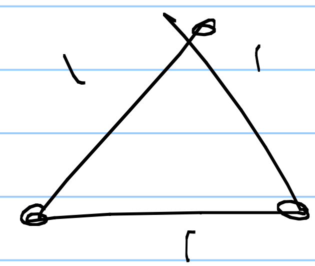
for any $u, v, w \in V$.



This inequality is always satisfied for edges + vertices drawn in the plane:

Ex:

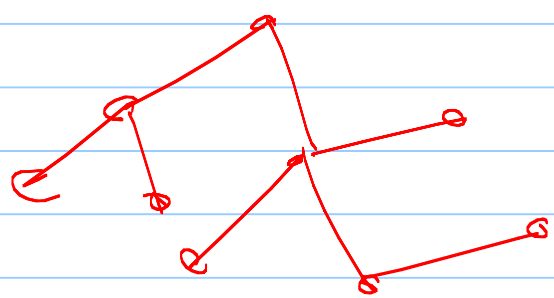




Thm: If the graph satisfies the triangle inequality, can compute a 2-approximation for TSP.

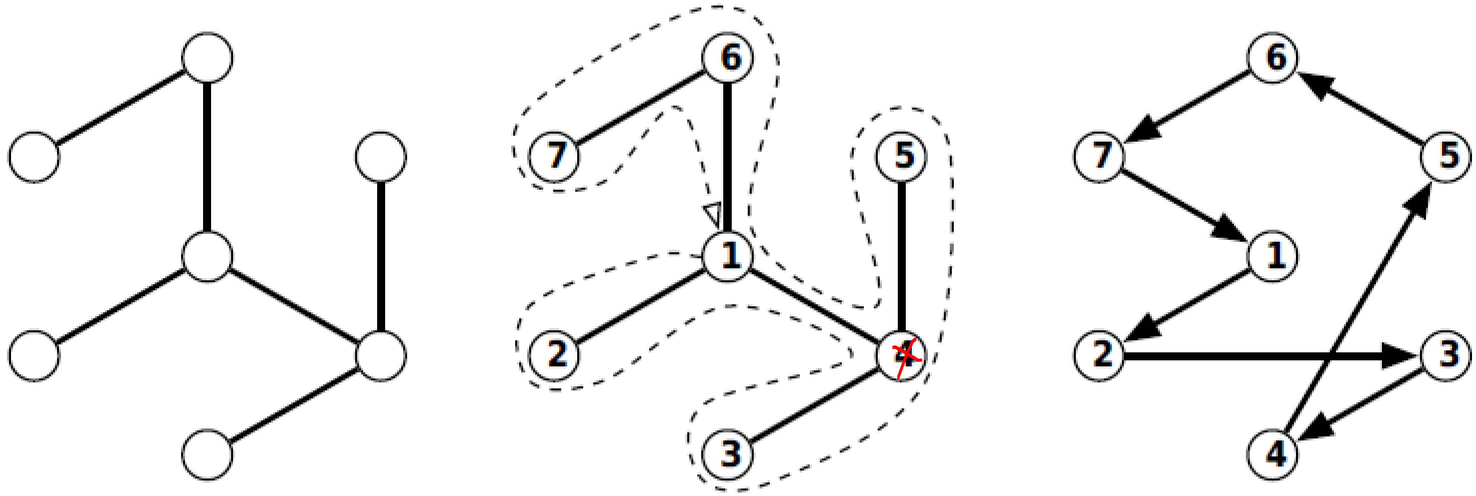
Idea: Use minimum spanning tree!

(Time to compute?)



$m \log n$ (?)

Idea



- Compute MST
- Get a DFS ordering $O(m+n)$
- Visit in this order

proof: Let OPT be cost of optimal tour.

Let MST be length of MST.

Our alg's cost: A

$$A = 2 \cdot MST$$

$$\leq 2 \cdot \underline{MST}$$

Since traverse each edge in tree (originally) twice, & then shortcut.

On the other hand,
deleting one edge from cycle makes a tree

So: optimal cycle is a tree (path) plus an edge,

$$MST \le OPT$$

$$\Rightarrow \quad A = 2 \cdot MST \le 2 \cdot OPT$$