

CS314: Algorithms

Homework 9

1. Moe is deciding how much Regular Duff and how much Duff Strong beer to order for his pub each week. Regular Duff costs Moe \$1 per pint and he sells it at \$2 per pint; Duff strong costs \$1.50 and he can sell it for \$3 per pint.

However, as part of a complex marketing scam, the Duff company will only sell a pint of Duff Strong for every two or more pints of Regular Duff that Moe buys. Further, due to past events (better left untold), Duff will not sell Moe more than 3000 pints per week total. Moe knows that he can sell however much beer he has.

Formulate the linear program for deciding how much Regular Duff and how much Duff Strong to buy, so as to maximize Moe's profit. Note: You don't need to solve it, just set it up!

2. For this problem, we define a salad as any combination of the following ingredients: (1) tomato, (2) lettuce, (3) spinach, (4) carrot, and (5) oil. Each valid salad must contain: (a) at least 15 grams of protein, (b) at least 2 and at most 6 grams of fat, (c) at least 4 grams of carbohydrates, (d) at most 100 milligrams of sodium. Furthermore, (e) you do not want your salad to be more than %50 greens by mass (measured in grams of each ingredient, where lettuce and spinach both count as greens). The nutritional contents of these ingredients per 100 grams are:

ingredient	energy (kcal)	protein (grams)	fat (grams)	carbs (grams)	sodium (milligrams)
tomato	21	0.85	0.33	4.64	9.00
lettuce	16	1.62	0.20	2.37	8.00
spinach	371	12.78	1.58	74.69	7.00
carrot	346	8.39	1.39	80.70	508.20
oil	884	0.00	100.00	0.00	0.00

Find a linear programming applet on the web (there are a ton of them!) and use it to make the salad with the fewest calories under the nutritional constraints described above. Describe your linear programming formula and the optimal solution (both the quantity of each ingredient and the total value). Please make sure to cite the page you use, and if possible even include a screenshot or printout of the setup/solution.

3. As mentioned in class, an *integer program* is a linear program with the additional constraint that the variables must take on only integer values (not any real number).

Prove that finding the optimal feasible solution to an integer program is NP-Hard.

Hint: Almost ANY NP-Hard decision problem can be formulated as an integer program. Pick your favorite, and reduce to show hardness!