

CS314: Algorithms

Homework 4

1. Give an algorithm to detect whether an undirected graph contains a cycle. If the graph contains a cycle, your algorithm should output one. (Note: not all of them - just one!) The running time of your algorithm should be $O(m + n)$ for a graph with n nodes and m edges.

2. Let G be a connected graph, and let T be a depth-first spanning tree of G rooted at some node v . Prove that if T is also a breadth-first spanning tree of G rooted at v , then $G = T$ (so G cannot contain any other edges).

3. Whenever groups of pigeons gather, they instinctively establish a pecking order. For any pair of pigeons, one pigeon always pecks the other, driving it away from food or potential mates. The same pair of pigeons always chooses the same pecking order, even after years of separation, no matter what other pigeons are around. Surprisingly, the overall pecking order can contain cycles - for example, pigeon A pecks pigeon B, which pecks pigeon C, which pecks pigeon A.
 - (a) Prove that any finite set of pigeons can be arranged in a row from left to right so that every pigeon pecks the pigeon immediately to its left. (Hint: think induction!)
 - (b) Suppose you are given a directed graph representing the pecking relationships among a set of n pigeons. The graph contains one vertex per pigeon, and it contains an edge ij if and only if pigeon i pecks pigeon j . Describe and analyze an algorithm to compute a pecking order for the pigeons, as guaranteed by part (a).